Anonymous

Bunker hedging with Expected Loss Control by buffered Probability of Exceedance and Conditional Value-at-Risk

Authors

1. Sun, Xiaolin, xlsun@shmtu.edu.cn, [Presenter], (School of Transport, Shanghai Maritime University, Shanghai, China)
2. Ding, Shuxin, jackietindd@gmail.com, (School of Automation, Beijing Institute of Technology, Beijing, China)

Abstract

Bunker fuel costs could account for 50–60 per cent of a ship's total operating cost in times of high fuel prices. The hedge against the volatility of bunker fuel price has contributed to shipping risk aversion. Despite the traditional minimum-variance hedging's simplicity, this method has obvious disadvantages since the risk measure does not distinguish between loss and profit and equally penalizes both. Thus, we address this problem by buffered Probability of Exceedance (bPOE)， conditional Value-at-Risk (CVaR) to control for the risk of shortfalls, compared with other objectives including minimum-drawdown deviation, minimum-standard deviation. We also study the methodology when solving the optimization problem to minimize bPOE by building the efficient frontiers with CVaR constraints. Our findings are superior to the standard minimum-variance methods, and can be partly explained by the effect of estimation error and model misspecification.

Keywords: *Bunker fuel; minimum-variance hedging; expected loss; buffered Probability of Exceedance; conditional Value-at-Risk; drawdown deviation.*

1. Introduction

As a subsystem in the global transportation and logistics network, shipping composes nearly 90 per cent of international trade. Consequently, the shipping industry is highly susceptible to the fluctuations of the international economy (Alizadeh, 2004). It is estimated that bunker costs may accounts for more than half of the whole operational expense to shipping lines (Notteboom and Vernimmen, 2009, Shi et al., 2013). Thus, bunker risk aversion has become one of the most important vital issues in shipping industry in decades (Wang et al. 2013).

At a tactical level, shipping companies purchase bunker derivatives in the futures or forward markets to control the volatile spot fuel prices (Menachof,2001, Plum et al., 2014, Ghosh et al., 2015). However, a lot of big losses in shipping industry occurred which make their strategies un-efficient. So there have been good reasons for management layer's judgement and their perception of risk. This unusual phenomenon is also found in the literature (Chng, 2009, Mirantes et al.,2012, Pedrielli, 2015). From a theoretical perspective, the traditional objective of hedging is minimizing variance (Ederington, 1979, Wang et al., 2015). However, minimum-variance hedging is optimal from a risk reduction perspective only when investors have quadratic utility or when returns are drawn from a multivariate elliptical distribution (Cao et al. 2010). In practice, neither of these assumptions is likely to hold. When investors have preferences over higher moments of returns, which is inconsistent with quadratic utility, variance is no longer an appropriate measure of risk since it ignores multivariate elliptical distribution. This leads to find new measures of risk and investors' motivation.

We address the problem of bunker hedging by minimizing conditional Value-at-Risk (CVaR), a quantile downside risk measure which is introduced by Rockafellar and Uryasev(2000, 2002). In the context of hedging, Harris and Shen (2006) developed minimum-VaR and minimum-CVaR hedge ratios, estimated non-parametrically and semi-parametrically using historical simulation. The non-elliptical nature of return is particularly important in the hedging context because while minimum-variance hedging unambiguously reduces portfolio variance, it can actually increase negative skewness and kurtosis, leading to portfolios that are risker when measured by VaR or CVaR than when measured by variance. Kavussanos and Dimitrakopoulos (2011) investigate the medium-term market risk of ocean going tanker vessel freight rates based on VaR, finding that non-parametric models perform best in estimating VaR. In this paper, we also investigate the optimal bunker hedging strategy with other different deviation measures: standard deviation, drawdown and the traditional minimum-variance methods.

In addition, we consider new variants of the bunker fuel hedge problem with downfall risks controlled by buffered Probability of Exceedance (bPOE), (Mafusalov and Uryasev, 2014-1, Uryasev, 2014-2). bPOE is a function closely related to the Probability of Exceedance (POE), which is the chance that the spot change amount is higher than the future change generated by the portfolio at least at one time period. bPOE is an extension of the so called Buffered Probability of Failure considered by Rockafellar and Royset (2010). We compare optimization problem statements in which risks are controlled by bPOE and by CVaR, respectively, and explore the important implication and the practical relevance of bPOE. The bPOE concept is introduced in the basic engineering context (reliability of component design), and this is the first exposure of this concept to the finance hedging optimization.

It is important to note that constraints on bPOE and CVaR are equivalent in the sense explained later on in the paper. However, the problems of minimizing bPOE and CVaR are quite different. Minimizing bPOE is intended to reduce the probability of an undesirable event. In this paper, the undesirable event is when portfolio hedging results $\boldsymbol{\gamma_t(f_{t+1}-f_t)}$ is below or only slightly above the change of spot price $(\boldsymbol{s_{t+1}-s_t})$ at some time moment. This event includes tail outcomes such that the average of the tail equals to the threshold. We conduct an empirical study demonstrating that the hedging problems with bPOE functions can be efficiently solved with convex and linear programming. The optimization was done with the Portfolio Safeguard (PSG) package (American Optimal Decisions, 2009). PSG provides compact and intuitive problem formulations and codes for solving risk management problems.

This paper is organized as follows. We start with methodology. In Sections 2.1 we give the hedging strategy background. Then we define conditional VaR and buffered Probability of Exceedance in Section 2.2. With the help of PSG optimization package, we present approaches for control risks by CVaR-deviation, standard deviation, drawdown, deviation and bPOE in Section 3. In Section 4, we illustrate our model to a bunker hedge problem, then minimizing different portfolio risk objectives subject to an average error constraint or without it; empirical results of bPOE and its efficient frontiers are discussed in Section 4.2. Section 5 provides concluding remarks and acknowledgements.

2. Methodology

2.1 Theoretical background

Assume that a ship owner tries to hedge bunker fuel risk exposures. Consider the interactions among the bunker spot and derivatives prices, minimizing the traditional variance of the hedge portfolio would produce hedging errors. And whether the conceptual hedging errors are economically relevant is an empirical question to answer.

At time t, the ship must determine an optimal derivative position to minimize the risk of the positions of the underlying assets at time $t+1$. And the risk of the hedged portfolio is measured by the variance of the hedged portfolio returns. Let s_t, f_t, be the prices of the spot and derivatives, and γ_t the hedge ratio at time t. Then, h_{t+1} is defined as the bunker hedge, and $h_t = (s_{t+1}-s_t)-\gamma_t(f_{t+1}-f_t)$. Moreover, the variance of the hedged portfolio at time $t+1$, $Var(h_{t+1})$, is given by $Var(h_{t+1}) = Var(s_{t+1}) + \gamma_t^2 Var(f_{t+1}) - 2\gamma_t Cov(s_{t+1}, f_{t+1})$.

By minimizing the variance, the OHR (Optimal Hedge Ratios) at time t, i.e., r_t, is determined by

$$\gamma_t^* = \frac{Cov(s_{t+1},f_{t+1})}{Var(f_{t+1})}. \tag{1}$$

2.2 Defination of CVaR and bPOE

Suppose a random variable L is the future loss (or the return with a minus sign) of some hedge. By definition, Value-at-Risk at level α is the α-quantile of L,

$$\mathrm{VaR}_\alpha(\mathrm{L}) = \inf\{\{z|\mathrm{F_L}(z) > \alpha\} \tag{2}$$

where F_L denotes Cumulative Distributions Function (CDF) of the random variable L. Conditional Value-at-Risk (CVaR) for a continuous distribution equals the expected loss exceeding VaR (Rockafellar and Uryasev,2002),

$$\mathrm{CVaR}_\alpha(\mathrm{L}) = \mathrm{E}[\mathrm{L}/\mathrm{L} \geq \mathrm{VaR}_\propto(\mathrm{L})]. \tag{3}$$

This formula justifies the name of CVaR as a conditional expectation. For general distributions, the definition is more complicated, and can be found in Rockafellar and Uryasev(2002).

There are two probabilistic characteristics associated with VaR and CVaR. The first characteristic is the Probability of Exceedance (POE), which equals 1 minus CDF,

$$\mathrm{p_z}(\mathrm{L}) = \mathrm{P}(\mathrm{L} > z) = 1 - \mathrm{F_L}(z). \tag{5}$$

By definition, CDF is the inverse function of VaR. The second probabilistic characteristic is called Buffered Probability of Exceedance (bPOE).

$$\bar{\mathrm{p}}_\mathrm{z}(\mathrm{L}) = \min_{\lambda \geq 0} \mathrm{E}[\lambda(\mathrm{L} - z) + 1]^+, \lambda \geq 0 \tag{6}$$

Formula (6) is considered as a property of bPOE, but it is convenient to use it as a definition, as in this paper (Uryasev, 2014). It has been proved that bPOE equals $1 - \alpha$ on the interval EL $< z <$ sup L, where α is an inverse function of CVaR, i.e., a unique solution of the equation

$$\mathrm{CVaR}_\alpha(\mathrm{L}) = z. \tag{7}$$

where sup L is the essential supremum1 of the random value L.

Therefore, bPOE equals the probability, $1 - \alpha$, of the tail such that CVaR for this tail is equal to z. The formula (7), to some extent, could be surprising; the expression does not immediately come across as a probability of some event and it is not obvious that the value belongs to the interval [0, 1] for any real value z.

At the point z = sup L, the solution of equation (7) may not be unique. For z = sup L, the smallest solution equals 1 − P (L = z). The formula (6) corresponds to this smallest solution

$$\bar{p}_{z=\sup L}(L) = \max\{1 - \alpha | CVaR_\alpha(L) = z\}. \tag{8}$$

So bPOE corresponds to the largest value for 1 − α.

The largest solution of equation (8) at the point z = sup L equals α = 1, which corresponds to the smallest value of 1 − α = 0, i.e.

$$0 = \min\{1 - \alpha | CVaR_\alpha(L) = z\}. \tag{9}$$

3. Controlling risks by CVaR-Deviation, Standard Deviation, Drawdown_Deviation and bPOE

We investigate the optimal bunker hedging strategy with the fowllowing different deviation measures: standard deviation, CVaR deviation, dawndown deviation, bPOE and Variance.

Minimize CVaR_Dev (minimizing 90%-CVaR deviation),where it equals to CVaR Deviation for Loss. It is similar with standard deviation, drawdown, and variance.

Then we minimize bPOE. The maximum loss function, $L = \max_{0\leq t\leq N} L_t$, depends upon a set of decision vectors, $x_1, ..., x_N$. Let us combine these vectors in one decision vector $\vec{x} = (x_1, ..., x_N)$. Let us consider a general linear loss function $L_t(\vec{x}) = (\vec{a}_t)^T \vec{x} + b_t$ with random coefficients $\vec{a}_t$. Then, (8) implies $\bar{p}_z(L)$=minE[λ(max L_t −z)+1]$^+$ =minE[λ max{$(\vec{a}_t)^T\vec{x}+b_t$ −z}+1]$^+$. λ≥0 0≤t≤N λ≥0 0≤t≤N.

The minimization problem for bPOE w.r.t. $\vec{x}$ can be written as follows,

$$\min \bar{p}_z(L)= \min E[\lambda \max\{(\vec{a}_t)^T\vec{x}+b_t -z\}+1]^+ \tag{10}$$

$\vec{x}$ $\vec{x}$,λ≥0 0≤t≤N = min E[max $\{(\vec{a}_t)^T \lambda\vec{x} + \lambda(b_t - z)\} + 1]^+$.
(11)

$\vec{x}$,λ≥0 0≤t≤N By replacing the term $\lambda\vec{x}$ with $\vec{y}$ in the last equation we get the optimization problem with respect to variables $\vec{y}$, λ, min E[max {(⃗a_t)^T ⃗y + λ(b_t − z)} + 1]$^+$. ⃗y,λ≥0 0≤t≤N (12)

Further, we suppose that coefficients, ⃗a_t, are random vectors with finite discrete dis- tribution, and the random loss function L^k_t (⃗x) = (⃗a^k_t)^T ⃗x + b^k_t has scenarios k = 1, ...K with probabilities p_k = 1/K. By using new variables ⃗y,λ we rewrite problem (12) as follows,

$$\min_{\vec{y},\lambda} \sum_{k=1}^{K} p_k \left[\max_{0\le t\le N} \left\{ \left(\vec{a}_t^k\right)^T \vec{y} + \lambda\left(b_t^k - z\right) \right\} + 1 \right]^+ \quad (13)$$

min $p_k u_k$ (14)

y,λ, ⃗u k=1

subject to

$$p_0^T y_0 - d\lambda \le 0$$

$$\lambda \ge 0, y_t \ge 0, t = 0, ..., N.$$

The objective in (13) is called partial moment with threshold -1 of the random function $L = \max_{0\le t\le N} \left\{ \left(\vec{a}_t^k\right)^T \vec{y} + \lambda\left(b_t^k - z\right) \right\}$. This objective function is a piecewise linear convex function w.r.t. the variables ⃗y and λ . The problem (13) can be reformulated as a Linear Programming (LP) problem. Let us introduce an additional vector of decision variables ⃗u = (u_1 , ..., u_K). Problem (13) is equivalent to the following LP,

$$\min_{\vec{y},\lambda,\vec{u}} = \sum_{k=1}^{K} p_k u_k \quad (15)$$

subject to

$$u_k \ge \left(\vec{a}_t^k\right)^T \vec{y} + \lambda\left(b_t^k - z\right) + 1, t = 0, ..., N, k = 1, ..., K,$$

$$p_0^T y_0 - d\lambda \le 0,$$

$$\lambda \ge 0, y_t \ge 0, t = 0, ..., N, u_k \ge 0, k = 1, ..., K.$$

$$L = S_{t+1} - \gamma_t f_{t+1}$$

$$MinbPOE(z, L) = p_y(L) \quad (16)$$

subject to

$$\gamma_t \geq 0$$

$$E(L) \geq d$$

Then $p_z(L) = \min_{\lambda \geq 0} E[\lambda(L - z) + 1]^+ = \min_{\lambda \geq 0} E[\lambda(os - hof - z) + 1]^+ = \min_{\lambda \geq 0} E[\lambda(os - z) - \lambda hof + 1]^+$. Replace $\lambda h = y$

$$\min_{h} p_z(L) = \min_{h,\lambda \geq 0} E[\lambda(os - z) - \lambda hof + 1]^+ \quad (17)$$

$$= \min_{y,\lambda \geq 0} E[\lambda(os - z) - yof + 1]^+$$

We suppose that the loss function $L = of - hos$ has scenarios k=1,…,K with probalitities pk=1/K. The model can be rewrite as follows:

$$\min_{y,\lambda} \sum_{k=1}^{K} p_k[\lambda(os - z) - yof + 1]^+ \quad (18)$$

Subject to

$$\sum_{k=1}^{K} p_k(\lambda os - yof) \geq \lambda d$$

$$\lambda \geq 0$$

$$y \geq 0$$

This problem can be reformulated as a Linear Programming (LP) problem. An additional vector of decision variables u=(u1,u2,…,uk). The problem is equivalent to the following LP.

$$\min_{y,\lambda,u} \sum_{k=1}^{K} p_k u_k \quad (19)$$

Subject to

$$u_k \geq \lambda(os - z) - yof + 1, k = 1, \dots, K$$

$$\sum_{k=1}^{K} p_k(\lambda os - yof) - \lambda d \geq 0$$

$$\lambda \geq 0$$

$$y \geq 0$$

$$u_k \geq 0, k = 1, \dots, K$$

4. Empirical study

4.1.Data

The data set includes weekly spot price for IFO 180 heavy oil, weekly settlement prices of 1-month futures contracts on IFO 180 at Singapore Exchange, which are collected from Bloomberg. The data are from January 2011 to January 2017. Table 1 reports a set of preliminary statistics for the returns of spot and future IFO 180. First, the same mean and standard deviation show the relatively high liquidity of spot and future markets. Second, the skewness and kurtosis statistics suggest that return distribution is left skewed and fat tailed in two time series. Third, the Jarque and Bera (J-B) test confirms the fat-tailed distributions in two markets by rejecting the null hypothesis of a Gaussian distribution at the 1% significance level. The Q-statistics for the autocorrelation of returns and squared returns show that the null hypothesis of non-autocorrelation is rejected for the two markets. Furthermore, the F-statistics of the ARCH test explain an ARCH effect in the spot and future IFO 180 markets.

Table 1 – Summary statistics of returns of spot and future IFO 180

	T	Mean (%)	Std. dev.	Skewness	Kurtosis	J-B test	LB Q(12)	ARCH(12)
RS	312	-0.147	0.109	-1.085	20.150	5161.8***	18.727 ***	10.197 ***
RF	312	-0.147	0.109	-1.081	20.082	5161.6 ***	18.728 ***	10.197 ***

4.2. Hedging Results

We estimate two groups: one has constraint that the average residual error is equal to zero and the other is without the constraint. Table 1 reports the sample performance of the eight different objectives with and without constraints hedging strategies for the bunker portfolios. In each table it reports the average estimated hedging ratio, objective value, loss, CVaR deviation, mean absolute deviation, standard deviation, VaR 99%. In all cases, the standard deviation is significantly reduced reflecting the relatively high correlations between the spot and futures series. In contrast with the diversification effects on standard deviation, in two of the eight cases, the loss of the hedge portfolio is more than the other six cases. The largest increase in hedge is the Drawdown method with the constraint. On average across the eight cases, standard deviation reduced by 5.293, but the loss increases by 10.28. The consequence is that in terms of CVaR, the reduction in standard deviation that arises from hedging is offset by an increase in mean absolute deviation in four of the eight cases and hence the reduction in CVaR is less than the reduction in standard deviation. In deed, in this case, minimum-variance hedging actually increases CVaR. By minimizing the drawdown deviation, we obtained the best values for all these considered eight risk measures. Minimization of CVaR deviation leads to good results, whereas minimization of standard deviation gives the worst level for the rest downside risk measures. And results with the constraint are not obviously better than without the constraint.

Table 2 – A comparison between different methods

Method	Constraint	HR	Loss	CVaR_Dev	Meanabs_Dev	St_Dev	VaR
CVaR_Dev	Yes	1.009	1.42e-14	0.865	0.226	0.500	-0.205
CVaR_Dev	No	1.011	6.99e-15	0.856	0.231	0.490	-0.092
St_Dev	Yes	0.999	16.222	9.019	2.763	5.296	4.360
St_Dev	No	1.002	16.223	8.927	2.738	5.287	5.353
Min-Variance	Yes	0.993	28.054	9.019	2.763	5.296	4.367
Min-Variance	No	1.002	26.311	8.927	2.738	5.282	5.354
DrawDown	Yes	1.001	0	24.143	1.423	16.221	-24.608
DrawDown	No	1	0.00001	23.986	1.345	16.233	-32.178

Notes: The table reports the average hedging ratio, Loss,

4.3. bPOE results

In order to solve the optimization problems, we use the Portfolio Safeguard package to calculate bPOE results. From equation (10) to equation (19), we show that with the fixed parameter z, coinciding parts of the efficient frontiers can be generated with parameter α in equation (15). Figure 1 gives a frontier by solving problem bPOE for a serios of values of parameter d in equation 18. The figure contains the calculated values for bPOE and the bound on average loss. The results suggest that bPOE minimization can be solved very efficiently.

5. Conclusion

Maritime bunker hedging is one of the most important issues in the shipping industry. The traditional minimum-variance method ignore the small probability however large loss which made a lot of shipping companies's hedging strategies un-effective. Therefore, we propose different objective models including CVaR, Standard deviation, drawdown and bPOE methods, taking average error into account. In addition, this paper presents several results about the equivalence of different efficient frontiers. Theoretically, the minimization problem for bPOE can be solved by building the efficient frontier with CVaR constraints and finding the inverse solution corresponding to the threshold. The study suggests that bPOE and CVaR minimization problems can be solved very efficiently for shipping risk management and portfolio optimization.

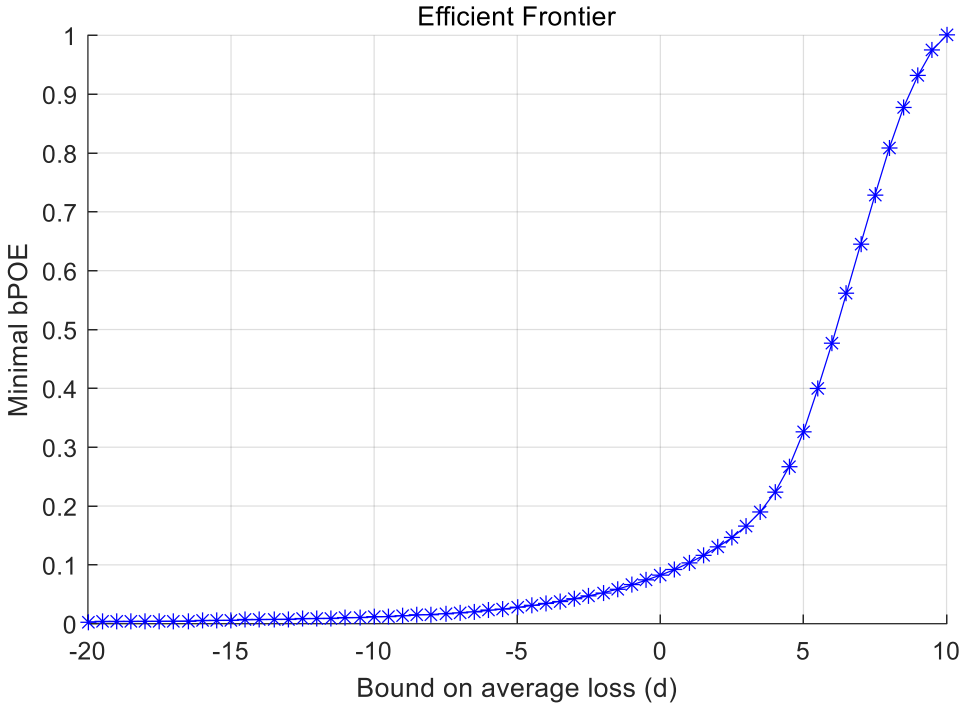

Figure 1 – Efficient frontier: bound on average loss vs. minimal bPOE

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References

Alizadeh, A. H., Kavussanos, M. G., and Menachof, D. A., 2004. Hedging against bunker price fluctuations using petroleum futures contracts: constant versus time-varying hedge ratios. Applied Economics, 36(12),1337–1353.

American Optimal Decisions (AORDA), Portfolio Safeguard (PSG), 2009.http:// www. aorda.com.

Cao, Z., Harris, R.D. and Shen, J., 2010. Hedging and value at risk: A semi-parametric approach. Journal of Futures Markets, 30(8), pp.780-794.

Chng, M.T., 2009. Economic linkages across commodity futures: Hedging and trading implications. Journal of Banking & Finance, 33(5), 958-970.

Ederington, L.H., 1979. The hedging performance of the new futures markets. The Journal of Finance, 34(1), 157-170.

Harris, R.D. and Shen, J., 2006. Hedging and value at risk. Journal of Futures Markets, 26(4), 369-390.

Ghosh, S., Lee, L. H., and Ng, S. H., 2015. Bunkering decisions for a shipping liner in an uncertain environment with service contract. European Journal of Operational Research, 244(3):792-802.

Mafusalov, A. and Uryasev, S., 2014, Buffered Probability of Exceedance: Mathematical Properties and Optimization Algorithms, Research Report, ISE Dept. University of Florida.

Menachof, D. A. and Dicer, G. N., 2001. Risk management methods for the liner shipping industry: the case of the bunker adjustment factor. Maritime Policy & Management, 28(2), 141–155.

Mirantes, A.G., Población, J. and Serna, G., 2012. Analyzing the dynamics of the refining margin: Implications for valuation and hedging. Quantitative Finance, 12(12), 1839-1855.

Notteboom, T.E. and Vernimmen, B., 2009. The effect of high fuel costs on liner service configuration in container shipping. Journal of Transport Geography, 17(5), 325-337.

Pedrielli, G., Lee, L. H., and Ng, S. H., 2015. Optimal bunkering contract in a buyer-seller supply chain under price and consumption uncertainty. Transportation Research Part E, 77,77–94.

Plum, C. E. M., Jensen, P. N., and Pisinger, D., 2014. Bunker purchasing with contracts. Maritime Economics & Logistics, 16,418–435.

Rockafellar, R. T. and J. O. Royset, 2010, On buffered failure probability in design and optimization of structures, Reliability Engineering and System Safety, 95(5), 499-510.

Rockafellar, R.T., and Uryasev, S., 2000, Optimization of Conditional Value-at-Risk, The Journal of Risk, 2(3), 21-41.

Rockafellar, R.T., and Uryasev, S., 2002, Conditional Value-at-Risk for General Loss Distributions, Journal of Banking and Finance, 26(7), 1443–1471.

Uryasev, S., 2014, Buffered Probability of Exceedance and Buffered Service Level: Definitions and Properties, Research Report ISE Dept., University of Florida.

Shi, W., Yang, Z. and Li, K.X., 2013. The impact of crude oil price on the tanker market. Maritime Policy & Management, 40(4), 309-322.

Wang, S., Meng, Q. and Liu, Z., 2013. Bunker consumption optimization methods in shipping: A critical review and extensions. Transportation Research Part E: Logistics and Transportation Review, 53, 49-62.

Wang, Y., Wu, C. and Yang, L., 2015. Hedging with Futures: Does Anything Beat the Naïve Hedging Strategy?. Management Science, 61(12), 2870-2889.