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A dynamic rescheduling and speed management approach for high-speed trains with uncertain time-delay

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ABSTRACT

This paper studies the problem of rescheduling trains with speed management in uncertain disruptions that lead to temporary speed restrictions (TSRs) in high-speed railway systems. The disruption uncertainties are that the coverage and speed limit value of TSRs change randomly over time, and the start and end times of TSRs are unknown in advance. First, a mixed-integer linear programming model is formulated to reduce the total train traveling times and improve the passenger comfortability. The solutions of the model provide simultaneously the optimal train rescheduling strategies and train speed control strategies. Second, a rolling horizon algorithm is applied in consideration of the disruption uncertainties. The rolling horizon algorithm updates and solves the model in every time horizon, according to the disruption information newly detected. Therefore, the train rescheduling orders and speed control orders can be adjusted according to the real-time situation to guarantee effectiveness. The rolling horizon approach is terminated automatically once the disruption ends. Finally, based on the practical data of the Beijing-Tianjin intercity high-speed railway line, numerical experiments are carried out to test the effectiveness and efficiency of the proposed approach.

1. Introduction

The high-speed railway provides a comfortable, efficient transportation mode. In China, by the end of 2020, the total mileage of high-speed railways in operation reached 37,900 km, accounting for 69% of the world [1]. The railway lines are constructed in a wild environment, subject to various disruptions [2]. Railway networks are complex systems. If not handled properly, small delays can be propagated and cause train delays in a large scale [3,4,39].

When disruptions occur, the train rescheduling problem is to minimize train delays by properly adjusting the train arrival and departure times for each passing station [5,40]. The disruption characteristics change over time, and the train rescheduling strategies need to be updated accordingly. However, the traditional rescheduling methods tend to generate solutions once and for all, and the random fluctuations of the disruption characteristics are ignored. Once the disruption characteristics change, the solutions calculated by the traditional methods can be infeasible. Different from the traditional methods, we focus on real-time train rescheduling

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problems. The proposed approach generates train rescheduling strategies according to the change of disruptions. Moreover, high-speed trains are operated by automatic train operation systems, which means that the train speed is controlled by machines. When disruptions happen and the normal train speeds are disturbed, it is necessary to develop effective speed control strategies for trains traveling through the affected area. The problem is termed the train speed management problem.

The real-time train rescheduling problem and the train speed management problem are treated separately in most of the literature. But the two problems are closely related, as interconnections do exist between the speed-related properties and the timetable-related properties [6–7]. There is a lack of effective methods to solve the two problems simultaneously. In this paper, we come up with methods to combine the two problems into an integrated one, which is termed the problem of real-time train rescheduling with speed management. The main contributions of this paper are as follows.

(1) We design an integrated mathematical model that is mixed-integer linear programming. The model aims to improve the punctuality and comfortability of trains while keeping operation safety. Moreover, the model is specially designed for the framework of a rolling horizon algorithm, which makes it possible to reschedule trains with speed management in consideration of the disruption uncertainties.

(2) The proposed approach is dedicated to generating real-time train rescheduling solutions, which can be used as the decision support tool. By now there are no decision support systems in China that help dispatchers reschedule trains with speed management. The rescheduling solutions make it easy to manage the train speeds for punctuality, operation safety, and comfortability.

2. Literature review

In recent years, train rescheduling problems have attracted extensive research. For the traditional train rescheduling methods, optimization models are designed based on the evaluation of the disruption characteristics, the models are solved offline to obtain the rescheduling commands once and for all. With the increase in train speed, train punctuality becomes more and more sensitive to disruption parameters that change randomly over time. The traditional methods gradually fail to meet the current demand for dynamic train rescheduling. Therefore, coming up with real-time train rescheduling methods to manage trains dynamically is necessary. First, we briefly introduce the literature on traditional train rescheduling methods, and then we introduce the literature on real-time train rescheduling methods. Because the train speed control strategy is also the focus of this paper, the literature in this area will also be introduced.

2.1. The train rescheduling problem

(1) Traditional train rescheduling methods

The traditional rescheduling methods believe the disruption parameters can be correctly estimated. For example, Zhan et al. [8] focused on rescheduling trains when the railway line is blocked and no trains can pass, and the blocking time is fixed in advance. To ensure the feasibility of the solution, the estimated track blocking time will be too conservative, which will be longer than the actual blocking time. Consequently, unnecessary train delays are caused. Veelenturf et al. [9] focused on large-scale train rescheduling in a situation where one or more tracks are blocked. Linear models were formulated with two objectives to minimize train delays and the number of canceled train services. Parts of rolling stock rescheduling problems were also considered in the model, but the model was solved only once, the solutions cannot be adjusted dynamically. Xu et al. [10] and Xu et al. [11] studied the train rescheduling problem in case of temporary speed restrictions. The disruption parameters such as the speed limit values, duration, and coverage were considered static in the model. Zhu and Goverde [12] designed rescheduling models that consider various train dispatching measures such as reordering, rerouting, retiming, flexible skipping, and flexible short turning of trains. Zhan et al. [13] focused on building models for rescheduling trains and optimizing passenger routes. The models were decomposed into two subproblems by the alternating direction method of the multipliers algorithm. The disruption considered is the blockage of a double-track railway line, whose lasting time in the model is also based on the estimation. For more related works about traditional train rescheduling, we can refer to [14], Gao et al. [15], Binder et al. [16], D'Ariano et al. [17], Zhu and Goverde [18] and Wang et al. [19].

(2) Real-time train rescheduling methods

The real-time train rescheduling methods are dedicated to adjusting train dispatching commands according to the change of disruption characteristics and restoring the normal train operation order in time after the disruption. For example, Zhan et al. [20] focused on rescheduling high-speed trains when one of the double tracks is partially blocked. The lasting time of the blockage is uncertain. A mixed integer linear programming model was designed and solved by a rolling horizon algorithm. Yin et al. [21] proposed a stochastic programming model to jointly optimize the passenger delay, the total train traveling times, and the train energy consumption. The stochastic passenger demand was modeled as Poisson distribution in the formulation, and the model is solved by an algorithm based on approximate dynamic programming. Bettinelli et al. [22] focused on real-time resolution of train conflicts, the proposed algorithm is based on executing a greedy approach repeatedly. The aim is to define actions that must be implemented to keep operation safety and reduce train delays. Fischetti and Monaci [23] designed a heuristic framework to guide the optimization software to solve the mixed-integer linear programming models for the real-time rescheduling of trains. Gao et al. [24] proposed an automatic rescheduling strategy, the constraints set of the optimization model is updated according to the latest feedback of fault handling information. Schoen and Koenig [25] formulated models by stochastic dynamic programming for the delay management problem on a single railway line, and the uncertainties of future delays are handled by simulating the potential recourse actions at later stations. Zhu and Goverde [26] focused on the dynamic train rescheduling problem when a railway is blocked. The models in Zhu and Goverde [12] were inherited and solved by the rolling horizon algorithm. Zhu and Goverde [27] proposed two approaches for rescheduling trains in situations where multiple disruptions occur unexpectedly. One is called the sequential approach which solves disruptions one by one, the other is called the combined approach which solves multiple disruptions simultaneously. For more work related to dynamic rescheduling, we can refer to Peng et al. [28] and Altazin et al. [29].

The real-time train rescheduling methods outperform the traditional train rescheduling methods in consideration of the disruption uncertainties. But in the literature, the dynamic rescheduling methods can be further improved by taking the speed management problem into account.

2.2. Rescheduling train with speed management

The train speed management problem focuses on calculating the optimal speed control strategies, and the calculation is based on the train traveling times which are the results of train rescheduling. Moreover, the train speed control strategies directly affect the train departure and arrival times involved in train rescheduling. Therefore, the train rescheduling problem and speed management problem are closely related. To find integrated solutions for the two problems is also widely concerned. Yang et al. [30] developed an integrated model to optimize the metro timetable and train speed profile. The model can be counted as a two-phase stochastic programming model. The temporal and spatial uncertainties of train mass, and the variabilities of the traction parameters including the tractive force, braking force, and basic traveling resistance are all considered in the model. The model is nonlinear therefore it was solved by a simulation-based genetic algorithm to find the near-optimal timetable and speed profiles for energy efficiency. Wang and Goverde [31] developed a multiple-phase optimal control model for the problem of train speed profile optimization. The general timetable constraints as well as the constraints related to signaling policies are considered. The developed method was dedicated to finding the balance between delay recovery and energy efficiency, which was testified under the delay and no-delay situations respectively. The work was further extended in Wang and Goverde [32], and a multi-train trajectory optimization method was developed for single-track lines. Three driving strategies in case of delay recovery, energy efficiency, and on-time driving were proposed for the selection of different objective functions in the model. Luan et al. [33] proposed integrated optimization models to manage the railway traffic for heterogeneous trains. The problem was formulated as a mixed-integer nonlinear programming problem (MINLP), which is reformulated as a mixed-integer linear programming problem (MILP) by linearizing the nonlinear constraints with piecewise affine functions. The models were solved by a self-designed two-step approach, to obtain a train rescheduling solution including the train routes, orders, departure and arrival times, and a train speed control solution including train speed trajectories. Luan et al. [34] extended the optimization models in Luan et al. [33] by considering the recycling of the train regenerative energy. Two objectives, namely delay recovery and energy efficiency were optimized utilizing the ε-constraint formulation and the weight-sum formulation. Wang and Goverde [6] developed an energy-efficient train timetabling method. The given train timetable was relaxed first to convert the departure and arrival times to time window constraints. A train profile optimization model was designed based on the time window constraints to find the optimal arrival and departure times. The proposed model was testified for the single-train profile optimization and multi-train profile optimization respectively. Yang et al. [35] developed an energy-efficient approach for metro trains to simultaneously calculate timetable and speed profiles under delay perturbations. The aim is to reduce net energy consumption and eliminate small train delays. The model formulated by integer programming is neither continuous nor convex, therefore it was solved by heuristic algorithms. Wang et al. [36] investigated the energy-efficient train timetabling and rolling stock circulation planning problem for metro lines. The energy-efficient train speed profiles were given in advance for different operation levels that involve different running times and dwelling times. The model was formulated to find the optimal operation levels and rolling stock circulation plans to reduce the overall energy consumption, moreover, the departure headways are calculated according to the passenger demands.

In summary, the previous studies on rescheduling trains with speed management tend to generate integrated solutions once and for

Table 1
Summary of the related works.

Publications	Considered disruptions/ disturbances	Consider disruption uncertainty	Rescheduling trains speed management	Solution flexibility	Model structure
Zhan et al. [20]	Railway blockage	Yes	No	Dynamic	EA-based MILP
Wang and Goverde [32]	Train delay	No	Yes	Non-dynamic	Optimal control model
Xu et al. [10]	Temporary speed restriction	No	Yes	Non-dynamic	AG-based MILP
Luan et al. [33]	Train delay	No	Yes	Non-dynamic	MILP, MINLP,
Zhu and Goverde [12]	Railway blockage	No	No	Non-dynamic	EA-based MILP
Yang et al. [35]	Train delay	No	Yes	Non-dynamic	MINLP
Zhu and Goverde [26]	Railway blockage	Yes	No	Dynamic	EA-based stochastic model
Zhu and Goverde [27]	Railway blockage	Yes	No	Dynamic	MILP
Zhang et al. [38]	Train delay	Yes	No	Dynamic	MILP
This work	Temporary speed restriction	Yes	Yes	Dynamic	MILP

Description for Table 1: MILP (Mixed-integer linear programming), MINLP (Mixed-integer nonlinear programming), EA-based (Event-activity based), AG-based (Alternative graph based).

all, the solutions lack flexibility and cannot be adjusted according to the change of disruptions. In this paper, we extend the previous research from the following two aspects. First, the proposed approach generates real-time solutions to dispatch trains with speed management, the solutions can be dynamically updated according to disruption data fluctuations. Second, real-time solutions for rescheduling trains with speed management in the situation of temporary speed restrictions are investigated, as the methodology for this situation is still lacking. The comparison between this paper and some typical works is listed in Table 1.

3. Problem description

3.1. Chinese high-speed railway infrastructure

Fig. 1 presents the infrastructures in Chinese high-speed rail systems. The direction from station 1 to station N is the downstream direction, while the opposite is the upstream direction. Double-track railway lines are applied. Train i heading in downstream direction runs on the downstream track, while train j heading in the upstream direction runs on the upstream track. Practically, downstream trains and upstream trains travel separately on different tracks. The downstream track is dedicated to downstream trains, the upstream track is dedicated to upstream trains. The railway switches are used by trains to switch tracks. For example, the downstream trains can use switches to switch to upstream tracks. In this paper, the trains do not switch tracks, because the TSRs only limit the train speeds, switching tracks is unnecessary for trains in this case from a practical perspective. Moreover, railway tracks are divided into block sections which can only be occupied by at most one train at any time. For instance, if train i occupies the block section s, other trains can only occupy this block section after train i leaves it. Therefore, trains can be well separated by block sections to keep operation safety.

Denote by $N^{set} = \{1, 2, \ldots, N\text{-}1, N\}$ the set of stations. As the downstream and upstream trains travel separately on different tracks, without loss of generality, we focus on rescheduling trains on the downstream track, see Fig. 2(a). Denote by $B^{set} = \{1, 2, 3, \ldots, B\text{-}1, B\}$ the set of block sections on the downstream track. Block section 1 is the first section in station 1, while block section B is the last section in terminal section N.

Block sections are classified into inbound sections, outbound sections, platform sections, and open-track sections, see Fig. 2(b) for details. The inbound sections, platform sections, and outbound sections are three kinds of sections that are covered by stations. In Fig. 2 (b), the train i first passes through the inbound section (i.e., block section s) to enter station 2, then dwells at the platform section (i.e., block section $s + 1$) for boarding and alighting, after which the train i travels through the outbound section (i.e., block section $s + 2$) when leaving the station 2. The open-track sections are the block sections that are not covered by any stations, e.g., block section $s + 3$.

Furthermore, the layouts of the stations can be illustrated in Fig. 3. Parallel tracks are equipped in the platform sections. In Fig. 3, track 0, track 1, and track 2 are the parallel tracks in station n in the downstream direction. Normally, track 0 is used by trains to pass through the station without stopping, other tracks are used by trains to stop for boarding and alighting. The number of parallel tracks equipped in different stations can be different. Denote by $Tr^{n,set} = \{0, 1, \ldots, Tr^n\}$ the set of parallel tracks in station n in the downstream direction. Table 2 and Table 3 list the notations in this paper.

3.2. Train speed control

As listed in Table 4, train speed is classified into six speed grades. Each speed grade ξ corresponds to a maximum speed V_ξ, for $\xi = 1$, 2, 3, 4, 5, 6. A train traveling at speed grade ξ means the speed of this train cannot surpass the value of V_ξ. The value of V_ξ is defined by the following equations.

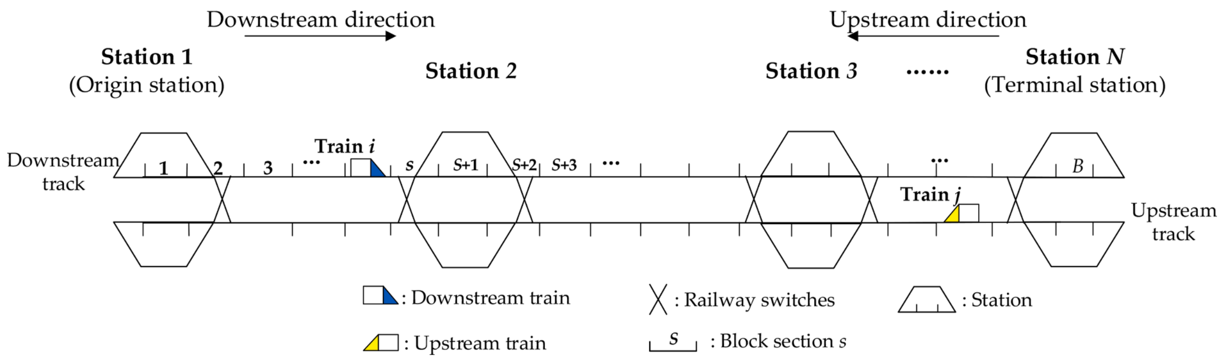


Fig. 1. Railway infrastructures.

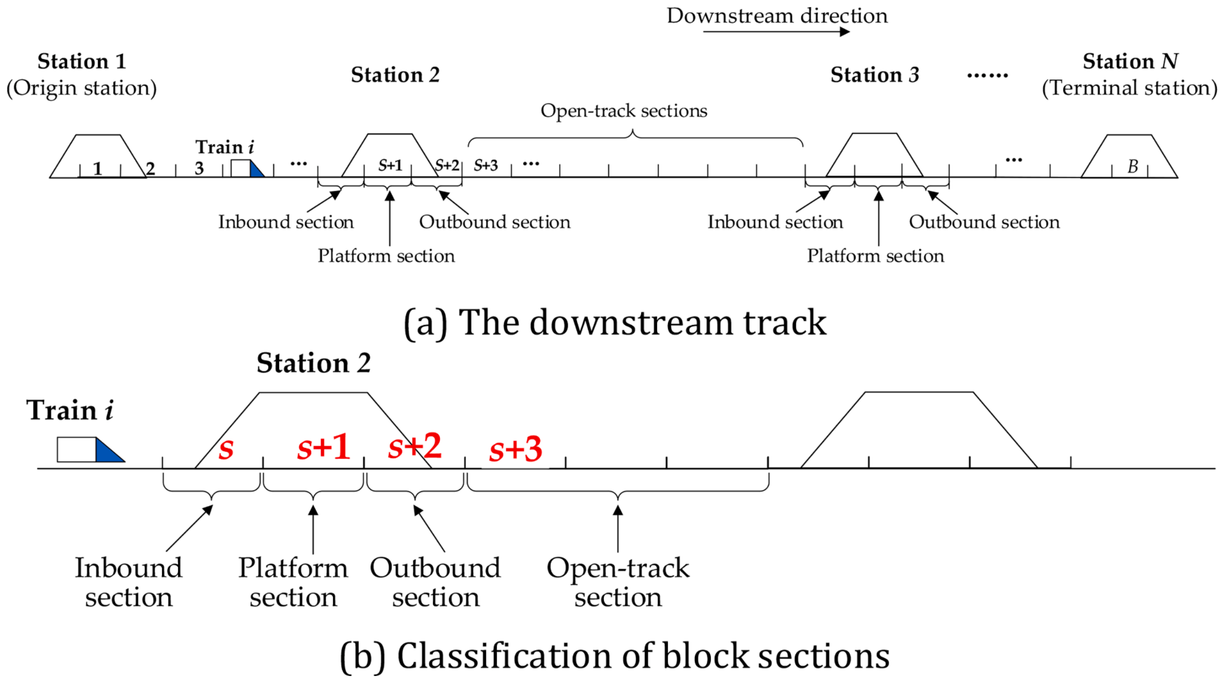


Fig. 2. The downstream track.

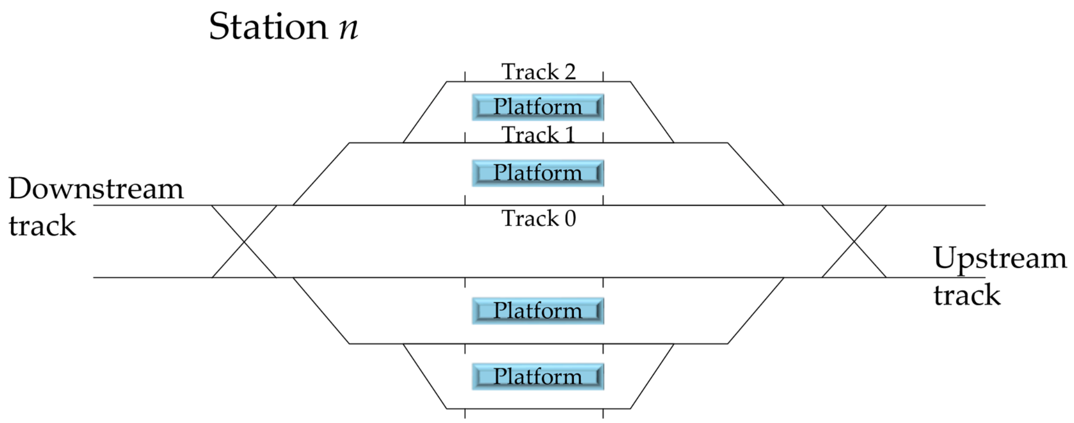


Fig. 3. Layouts of the parallel tracks in platform sections in the downstream direction.

$$\begin{cases} V_1 = 120\ km/h \\ V_2 = 160\ km/h \\ V_3 = 200\ km/h \\ V_4 = 250\ km/h \\ V_5 = 300\ km/h \\ V_6 = 350\ km/h \end{cases} \tag{1}$$

In this paper, trains cannot change speed grades within a block section, but the speed grades of a train in different block sections can be different. Trains in each block section travel at only one speed grade, therefore, the top speeds of each train in each block section are determined by the speed grades. Fig. 4 illustrates the space–time diagram of a train, the train travels from block section 1 to 7, the horizontal and vertical axes represent time and space respectively, the colors of the trajectory denote the speed grades of this train traveling in each block section. Hereinafter, the space–time diagram is termed train timetable, and the term is widely used in literature. When disruptions happen and make train delays inevitable, the necessity is to reschedule trains by reoptimizing the timetable to reduce total train delays.

In this paper, the lengths of the block sections are set as $L = 1360$ m, the value is commonly used in a typical high-speed railway design, see [10]. If the train travels at the biggest speed grade that is grade 6, the least traveling time to run through a block section is

Table 2
Indexes and sets.

Symbol	Description
i, j	Train index.
n	Station index.
k, v	Block sections index.
c	The index of the parallel tracks in a station.
ξ	The index of train speed grades. With a small abuse of notation, ξ also denotes the number of unoccupied block sections ahead of the train traveling on open tracks.
δ	The index of time horizons.
R_δ	The δ-th time horizon.
T^{set}	The set of trains.
B^{set}	The set of block sections, $B^{set} = \{1, 2, 3, \ldots, B\text{-}1, B\}$.
N^{set}	The set of stations, $N^{set} = \{1,2,3, \ldots, N\text{-}1, N\}$.
$Tr^{n,\ set}$	The set of parallel tracks in station n in the downstream direction, $Tr^{n,\ set} = \{0, 1, \ldots, Tr\}$.
$B^{n,\ in}$	The set of inbound sections in station n in the downstream direction.
$B^{n,\ out}$	The set of outbound sections in station n in the downstream direction.
$B^{n,\ p}$	The set of platform sections in station n in the downstream direction.
$B_{<n,\ n+1>}$	The set of open track sections between station n and $n + 1$.
B_{TSR}	The set of block sections influenced by temporary speed restrictions.
$N_{i,\ d}$	The set of stations where train i plans to dwell in the origin schedule.
E	The set of train speed grades in the normal situation, $E = \{1, 2, 3, 4, 5, 6\}$.
E_δ	The set of train speed grades in the area affected by temporary speed restrictions in the δ-th time horizon R_δ.

Table 3
Parameters.

Symbol	Description
$p_{i,k,t(\delta)}$	0–1 parameters. If train i occupies block section k at time $t(\delta)$, we have $p_{i,k,t(\delta)} = 1$, otherwise $p_{i,k,t(\delta)} = 0$.
$q_{i,k,t(\delta)}$	0–1 parameters. If train i has passed block section k at time $t(\delta)$, we have $q_{i,k,t(\delta)} = 1$, otherwise $q_{i,k,t(\delta)} = 0$.
η_s	The basic running time, see (2).
Δt_ξ	The additional running time, see (3).
V_ξ	Top speed when the train travels at speed grade ξ.
L	The length of a block section.
$A_{i,k,t(\delta\text{-}1)}$	The arrival time of train i at block section k given by timetable $TB_{\delta\text{-}1}$, the $TB_{\delta\text{-}1}$ is calculated by solving the model in the (δ-1)-th time horizon $R_{\delta\text{-}1}$.
$G_{i,k,t(\delta\text{-}1)}$	The speed grade of train i at block section k given by $TB_{\delta\text{-}1}$.
D^{min}	The minimum dwelling time of trains at a station.
D^{max}	The maximum dwelling time of trains at a station.
$y_{i,n}$	The planned departure time of train i at station n in the origin schedule.
W_1	The start time of the temporary speed restrictions.
h_d	The departure headway between two trains leaving from the same station consecutively in the downstream direction.
h_a	The arrival headway between two trains arriving at the same station consecutively in the downstream direction.
h_{da}	The departure-arrival headway, the minimum time difference between the previous train leaving the track and the next train entering the track in a platform section.
$t(\delta)$	The start time of the δ-th time horizon.
H	The duration of a time horizon.
r	The TSRs are detected every time r to decide whether to end the rolling horizon algorithm.
TB_δ	Timetable generated for the time horizon R_δ by solving the model.
M	A constant that is big enough in the model.
Q_1, Q_2	The weights of the objective functions.

Table 4
The train speed grades.

Speed grade ξ	$\xi = 1$	$\xi = 2$	$\xi = 3$	$\xi = 4$	$\xi = 5$	$\xi = 6$
Top speed (km/h)	V_1	V_2	V_3	V_4	V_5	V_6
Δt_ξ (s)	27 s	17 s	11 s	6 s	3 s	0 s

about $L/\widetilde{v}_6 = 14$ s, which is termed as the basic running time, and indicated by the notation $\eta_s = 14$ s thereinafter, see

$$\eta_s = \frac{L}{V_6} = 14s \tag{2}$$

Moreover, we define the additional running time Δt_ξ by

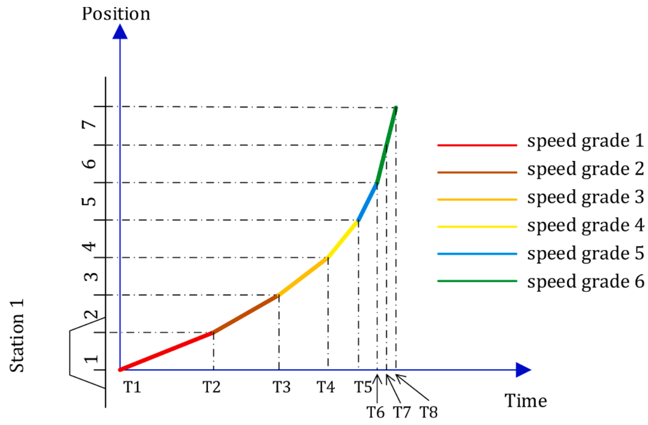


Fig. 4. The space–time diagram of a train.

$$\Delta t_{\xi} = \left\lceil \frac{L}{V_{\xi}} - \eta_S \right\rceil \tag{3}$$

The operator $\lceil \bullet \rceil$ is for rounding up. Therefore, if a train travels at speed grade ξ, the least traveling time is $(\eta_s + \Delta t_\xi)$ to pass through a block section. The values of Δt_ξ are listed in Table 4.

Moreover, to avoid rear-end collisions, a safety distance between any two trains traveling consecutively needs to be guaranteed in case of emergency braking. The safety distances are measured by the number of unoccupied block sections between the trains. The faster the following train, the longer braking distance is needed, therefore more unoccupied block sections should be kept between the two traveling trains. To be specific, in Fig. 5, train j travels behind train i in open-track sections between station n and station $n+1$. If train j travels at speed grade ξ, then ahead train j at least ξ unoccupied block sections are required to keep a safety distance from train i.

In practical situations, the speed grades of a train can change as the train travels through different block sections, especially when traveling through temporary speed restriction areas, the safety distances need to be adjusted dynamically.

3.3. The uncertainties of the temporary speed restrictions

The characteristics of the TSRs change randomly as time goes on. For example, in Fig. 6(a) at time $\widetilde{T}_1$ the train speeds are limited to grade 1 at the block sections highlighted by red color, then the limitation goes to grade 3 at time $\widetilde{T}_2$, see Fig. 6(b), finally it changes at time $\widetilde{T}_3$ to grade 5, see Fig. 6(c). As we can see, the coverage of the affected block sections also changes. The Fig. 6(d) presents the fluctuation of the speed limit value caused by TSRs, the change times $\widetilde{T}_1$, $\widetilde{T}_2$, $\widetilde{T}_3$, $\widetilde{T}_4$, and the speeds $\widetilde{V}_1$, $\widetilde{V}_2$, $\widetilde{V}_3$ are all random variables.

To handle the uncertainties, there are two popular methodologies in literature, namely the stochastic programming methods, and the rolling horizon algorithm. The models for stochastic programming are based on the prediction of disruptions. In practice, the TSRs fluctuate randomly, and it is impossible to obtain correct prediction results every time. The incorrect prediction results can lead to ineffective rescheduling solutions. Therefore, we prefer the rolling horizon algorithm. The algorithm updates the parameters in the model in real time. The model is solved for effective, real-time rescheduling solutions in each time horizon. Moreover, rescheduling

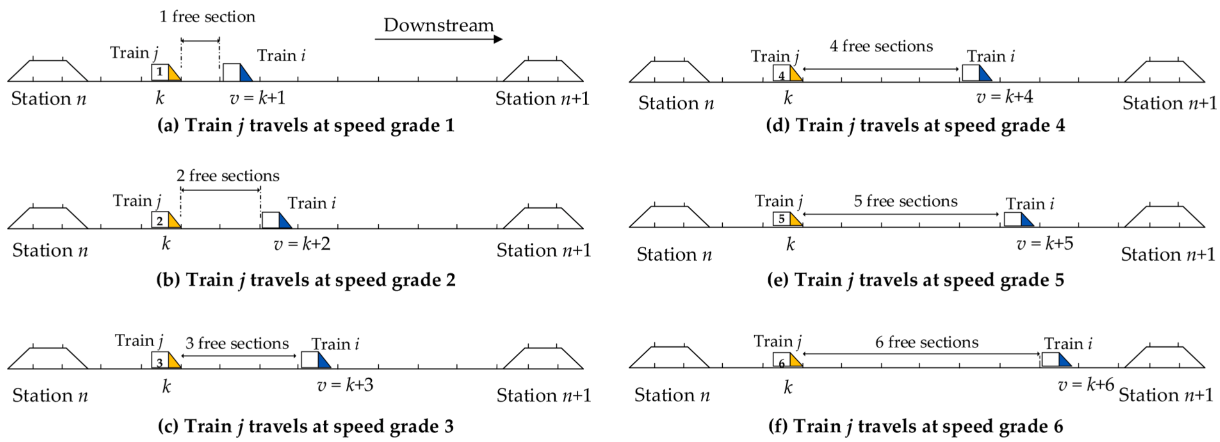


Fig. 5. The correlation between the safety distances and the train speed grades.

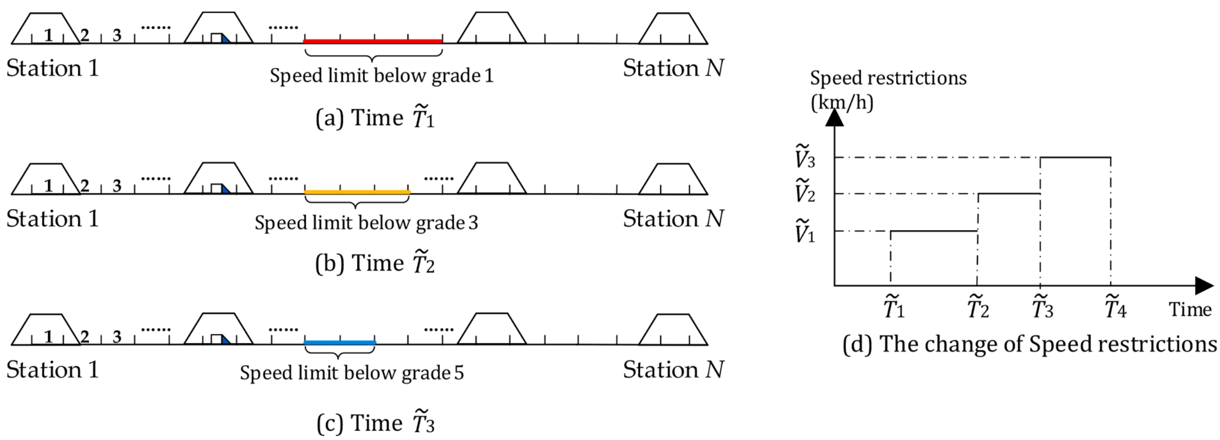


Fig. 6. The uncertainties of temporary speed restrictions.

trains and managing train speeds are two problems, in this paper by applying the rolling horizon algorithm, the two problems are integrated.

The TSRs usually make the origin timetable infeasible, in this case, the train delays are hard to avoid, and the main purpose of train dispatchers is to reduce train delays. Therefore, in this paper train delays are allowed, and we design a MILP model that aims to reduce train delays. We make the following assumptions.

(a) The temporary speed restrictions only limit train speed, but do not cause railway blockage. Trains can still travel through the TSRs area at relatively low speeds.
(b) The TSRs are calculated in real time according to the feedback of the sensors equipped along the railway tracks that monitor the factors such as wind speed, the floods that affect the train speed. The TSRs can be trusted. The algorithm judges whether to continue the calculation at the beginning of each time horizon according to the feedback. If the feedback show that no more disruptions are detected, the algorithm ends naturally.

4. Rolling horizon algorithm

To formulate a MILP model that can be applied in a rolling horizon framework, two kinds of 0–1 parameters are defined. As shown in Fig. 7, train i heads to terminal station N. At the moment $t(1)$, train i travels at the block section k, then we have the binary parameters

$$p_{i,k,t(1)} = \begin{cases} 1, & \textit{if train i occupis block section k at moment } t(1) \\ 0, & \textit{otherwise}, \end{cases} \tag{4}$$

Moreover, we define the binary parameter as follows.

$$q_{i,k,t(1)} = \begin{cases} 1, & \textit{if train i has passed the block section k at time } t(1) \\ 0, & \textit{otherewise} \end{cases} \tag{5}$$

In Fig. 7, at time $t(1)$, train i has passed the block sections before block section k, therefore we have $q_{i,v,t(1)} = 1$, for $v < k$, moreover we have $q_{i,v,t(1)} = 0$, for $v \geq k$.

The two 0–1 parameters are to specify the train positions. In this paper, the proposed MILP model is solved in a rolling horizon framework. The flow chart and pseudocode of the rolling horizon algorithm is presented in Fig. 8.

After initializing the static parameters, the algorithm is triggered when the TSRs are detected at time W_1. The first time horizon R_1 begins at $t(1) = W_1$. At the beginning of each time horizon, the dynamic parameters, i.e., the train positions $p_{i,k,t(\delta)}$ and $q_{i,k,t(\delta)}$, the set of affected block sections B_{TSR}, and the speed limits E_δ are updated. Then, the updated model is solved and a new timetable is obtained.

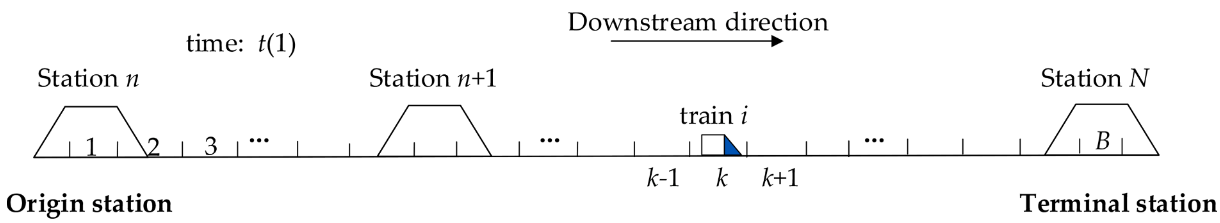
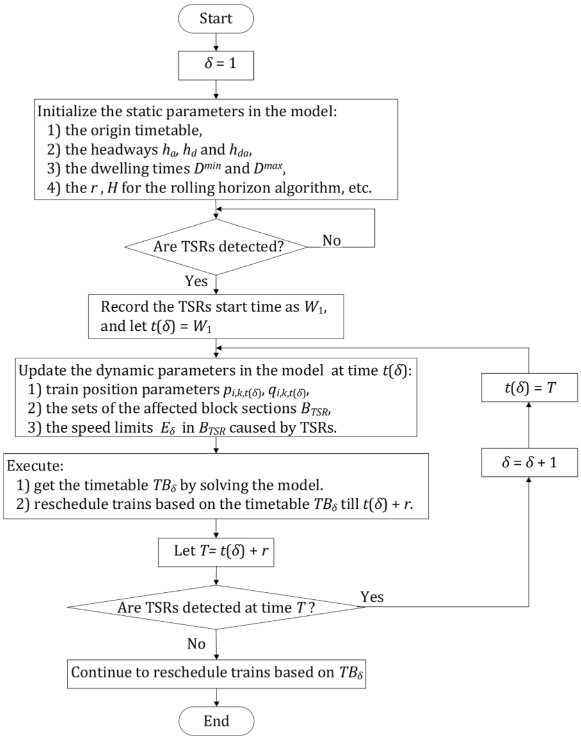


Fig. 7. Train i occupies block section k at time $t(1)$.



(1) The flow chart of the rolling horizon algorithm

```
Start
Step 1: let δ = 1.
Step 2: Initialize the static parameters, including the origin timetable, the headways h_a, h_d
    and h_da, the dwelling times D^min and D^max, and the parameters r and H, etc.
Step 3:
  While True
   IF TSRs are detected
     Step 3.1: record the start time of TSRs as W_1.
     Step 3.2: let t(δ) = W_1.
     Go to step 4.
     Break
   End IF
  End While
Step 4:
  While True
    Step 4.1: Update the dynamical parameters at time t(δ), including the train positions
        p_i,k,t(δ) and q_i,k,t(δ), the sets of the affected block sections B_TSR, and the speed limits E_δ.
    Step 4.2: Initialize the model based on the static parameters and dynamic parameters
        obtained in step 2 and step 4.1, respectively.
    Step 4.3: Get the timetable TB_δ by Solving the model newly obtained in step 4.2.
    Step 4.4: Reschedule the trains according to TB_δ till time t(δ)+r.
    Step 4.5: Let T = t(δ)+r.
    Step 4.6:
       IF TSRs are canceled at time T
       Go to step 5.
         Break
       End IF
  End While
Step 5: Continue to reschedule the trains based on TB_δ.
End
```

(2) The pseudocode to illustrate the rolling horizon algorithm

Fig. 8. The flow chart and pseudocode of the rolling horizon algorithm.

Trains are rescheduled according to timetables newly obtained in each time horizon.

To be specific, the δ-th time horizon R_δ begins at $t(\delta)$. At time $t(\delta)$, we assume that the TSRs do not change for the next H minutes. The assumption is reasonable for two reasons. First, the H is relatively small and can be decided by workers according to the real situation. Second, in practice, the speed limitations caused by TSRs are also decided by workers, and the frequency of changing TSRs will not be too fast. Therefore, the model updated at $t(\delta)$ is solved to generate the timetable TB_δ that can be used to reschedule trains during the δ-th time horizon.

See Fig. 9, before the end of R_δ, at time $t(\delta+1) = t(\delta) + r$ ($r < H$), we judge if the TSRs are ended to decide whether to continue the algorithm. If the TSRs are still effective at time $t(\delta+1)$, a new time horizon $R_{\delta+1}$ is triggered, and the model is solved based on the updated parameters to generate timetable $TB_{\delta+1}$ for time horizon $R_{\delta+1}$. The r and H can be defined by the train dispatchers. Every time r, a new time horizon is triggered if the TSRs are still valid. However, if the TSRs are ended, no more time horizons need to be triggered,

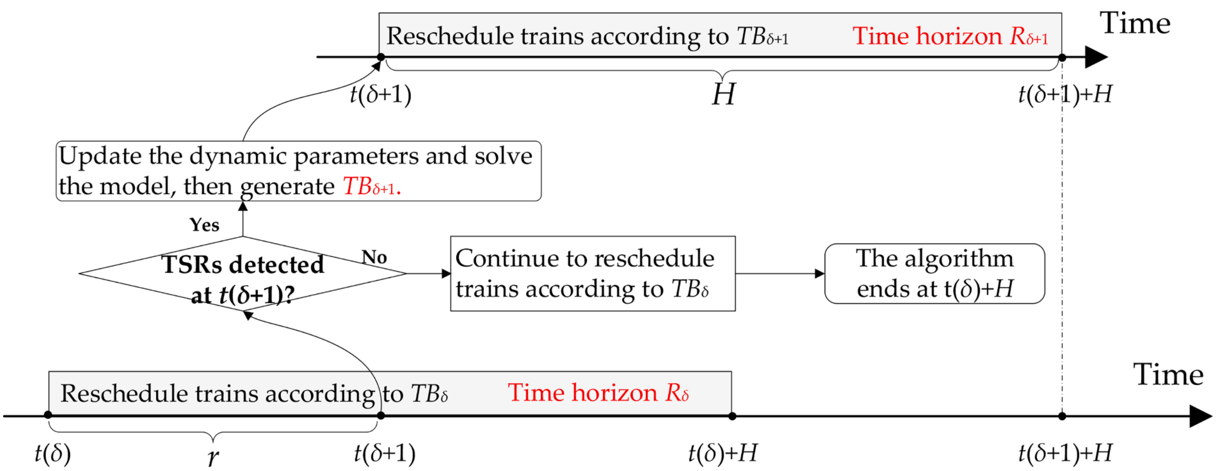


Fig. 9. Transition of the time horizons.

therefore the algorithm is ended. In this paper, the cost of time used to update the parameters and solve the model can be ignored.

5. Model formulation

We illustrate the model of the δ-th time horizon R_δ, which is updated according to the dynamic parameters $p_{i,k,t(\delta)}$, $q_{i,k,t(\delta)}$ and $g_{i,k,t(\delta)}$, and the timetable $TB_{\delta\text{-}1}$ calculated in the previous time horizon $R_{\delta\text{-}1}$. The decision variables are listed in Table 5, and the parameters of the model are given in Table 2 and Table 3.

(1) Constraints of train running time in block sections

To constrain the lower bounds of the train running times at each block section, we use the following inequalities.

$$a_{i,k+1} - a_{i,k} \geq \eta_s + \sum_{\xi=1}^{E} \beta_{i,k,\xi} \Delta t_\xi - q_{i,k,t(\delta)} M, \ i \in T^{set}, \ k \in B^{set} \tag{6}$$

The variable $a_{i,k}$ denotes the time when train i arrives at block section k. The model only limits the traveling times in the block sections that are before the trains at time $t(\delta)$. If train i has passed block section k, we have $q_{i,k,t(\delta)} = 1$ and the (6) becomes invalid. The binary variable $\beta_{i,k,\xi}$ denotes whether the speed grade of train i at block section k is ξ. If $q_{i,k,t(\delta)} = 0$ is true, then the lower bound of the block section traveling time is decided by the speed grade of train i. For example, if the speed grade of train i is grade 1, then we have $\beta_{i,k,1} = 1$ and the lower bound is $\eta_s + \Delta t_1$ according to (6).

Moreover, the upper bounds of the block section traveling times in the open-track sections are constrained by (7) and (8). The (7) provides the upper bound when the train speed grade is bigger than grade 1, and the (8) is for the upper bound when the speed grade is equal to grade 1. The upper bounds are used to prevent the train from stopping in the open-track sections.

$$a_{i,k+1} - a_{i,k} \leq \eta_s + \sum_{\xi=2}^{E} \beta_{i,k,\xi} \Delta t_{\xi-1} + M\beta_{i,k,1} + q_{i,k,t(\delta)} M, \ i \in T^{set}, \ k \in B_{<n,n+1>}, \ n \in N^{set}, \ n \neq N \tag{7}$$

$$a_{i,k+1} - a_{i,k} \leqslant \eta_s + 2 \cdot \Delta t_1 + M\left(1 - \beta_{i,k,1}\right) + q_{i,k,t(\delta)} M, \quad i \in T^{set}, \ \ k \in B_{<n,n+1>}, \ \ n \in N^{set}, \ n \neq N \tag{8}$$

(2) Constraints related to the previous time horizon

To make the solutions in the current time horizon feasible, the arrival times obtained in the previous time horizons are to be considered. The $A_{i,k+1,t(\delta\text{-}1)}$ is the planned arrival time of train i at block section $k+1$ in the previous time horizon $R_{\delta\text{-}1}$. If $p_{i,k,t(\delta)} = 1$ is true, the arrival time $a_{i,k+1}$ at the block section k in the δ-th time horizon is no less than $A_{i,k+1,t(\delta\text{-}1)}$, see (9).

$$a_{i,k+1} \geq \sum_{k=1}^{N-1} p_{i,k,t(\delta)} A_{i,k+1,t(\delta-1)}, k \in B^{set}, \ i \in T^{set} \tag{9}$$

At the beginning of each time horizon, a new timetable is generated, and the timetable obtained in the previous time horizon is abandoned. The (9) is to make the switch from the previous timetable to the current timetable reasonable.

The parameter $G_{i,k,t(\delta\text{-}1)}$ is the planned speed grade of train i at block section k in the previous time horizon. See Fig. 10, the block sections k and $k+1$ are two adjacent sections, and train i travels from block section k to $k+1$. Note that the $t(\delta)$ is the beginning time of R_δ. If at time $t(\delta)$ the train i occupies block section $k+1$, i.e., $p_{i,k,t(\delta)} = 1$, the change of speed grades from block section k to block section $k+1$ are less than 1, see (10). The (10) limits the change of the speed grades between the current time horizon and the previous time horizon, which is to avoid sharp train acceleration or deceleration.

$$\left| g_{i,k+1} - G_{i,k,t(\delta-1)} \right| \leq 1, \ if \ p_{i,k,t(\delta)} = 1 \tag{10}$$

The (10) can be reformed as the following equalities.

$$\begin{cases} g_{i,k+1} - G_{i,k,t(\delta-1)} \geq -1 - \left(1 - p_{i,k,t(\delta)}\right) M \\ g_{i,k+1} - G_{i,k,t(\delta-1)} \leq 1 + \left(1 - p_{i,k,t(\delta)}\right) M \end{cases}, \ k \in B^{set}, \ i \in T^{set} \tag{11}$$

(3) Dwelling time constraints

The (12) is for the lower bound of the train dwelling time at a station.

Table 5
Decision variables in the model in δ-th time horizon.

Symbol	Description
$a_{i,k}$	The arrival time of train i at block section k.
$g_{i,k}$	The speed grade of train i at block section k.
$\mu_{i,k}$	Binary variable to denote whether the speed grades of train i at block section k and $k+1$ are the same. If $g_{i,k} \neq g_{i,k+1}$, we have $\mu_{i,k} = 1$, otherwise we have $\mu_{i,k} = 0$.
$\beta_{i,k,\xi}$	Binary variable. If train i travels at block section k with speed grade ξ, we have $\beta_{i,k,\xi} = 1$, otherwise, $\beta_{i,k,\xi} = 0$.
$D_{i,n}$	Binary variable. If train i dwells at station n, we have $D_{i,n} = 1$, otherwise, $D_{i,n} = 0$.
$d_{i,k,c}$	Binary variable. If train i uses the c-th track at the platform section k, we have $d_{i,k,c} = 1$, otherwise $d_{i,k,c} = 0$.
$x_{i,k,1}$	Binary variable. If train i enters block section k after the time $t(\delta)$, we have $x_{i,k,1} = 1$, otherwise $x_{i,k,1} = 0$.
$x_{i,k,2}$	Binary variable. If train i enters section k before the time $t(\delta) + H$, we have $x_{i,k,2} = 1$, otherwise $x_{i,k,2} = 0$.
$\lambda_{i,j,n}$	Binary variable. If train i leaves from station n before train j, we have $\lambda_{i,j,n} = 1$, otherwise, $\lambda_{i,j,n} = 0$.

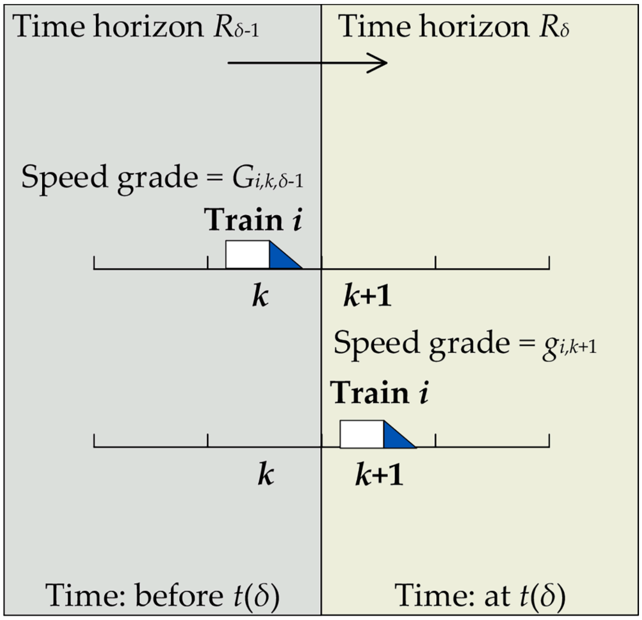


Fig. 10. The transition of train speed grades.

$$a_{i,k+1} - a_{i,k} \geq D^{min} + \left(D_{i,n} - 1\right)M - q_{i,k,t(\delta)}M,\ k \in B^{n,p},\ i \in T^{set},\ n \in N^{set} \tag{12}$$

$$a_{i,k+1} - a_{i,k} \leq D^{min} + D_{i,n}M + q_{i,k,t(\delta)}M,\ k \in B^{n,p},\ i \in T^{set},\ n \in N^{set} \tag{13}$$

If $q_{i,k} = 1$ is true, the inequalities become invalid due to the big-M structure. The binary variable $D_{i,n}$ denotes whether train i stops at station n. If $q_{i,k,t(\delta)} = 0$ and $D_{i,n} = 1$ are true, according to (12) the lower bound of dwelling time is D^{min}. However, if $D_{i,n} = 0$ that means train i passes through station n without dwelling, the traveling time on the platform section k is less than D^{min}, see (13). The (14) is for the upper bound of dwelling time. If $D_{i,n} = 1$ and $q_{i,k,t(\delta)} = 0$ are true, the dwelling time is less than D^{max}.

$$a_{i,k+1} - a_{i,k} \leq D^{max} + \left(1 - D_{i,n}\right)M + q_{i,k,t(\delta)}M,\ i \in T^{set},\ k \in B^{n,p},\ n \in N^{set} \tag{14}$$

(4) Station capacity constraints

The (15) and (16) are to decide which track to be used by trains in a station. The layouts of the stations can be found in Fig. 3. The $d_{i,k,0}$ denotes whether the track 0 is used by train i. According to (15), if train i passes through station n without dwelling, i.e., $D_{i,n} = 0$ and $q_{i,k,t(\delta)} = 0$, we have $d_{i,k,0} = 1$ which means track 0 is to be used. In fact, track 0 is only for trains that pass through the station without dwelling.

$$0.5 - d_{i,k,0} - Mq_{i,k,t(\delta)} \leq D_{i,n} \leq 1.5 - d_{i,k,0} + Mq_{i,k,t(\delta)},\ i \in T^{set},\ k \in B^{n,p},\ n \in N^{set} \tag{15}$$

According to (16), if $q_{i,k,t(\delta)} = 0$ and $D_{i,n} = 1$ are true, we have (1 - $d_{i,k,0}$) = 1 which means that $d_{i,k,0} = 0$ and train i is to pick a track other than track 0 to dwell due to (17).

$$-0.5 + \left(1 - d_{i,k,c}\right) - Mq_{i,k,t(\delta)} \leq D_{i,n} \leq 0.5 + \left(1 - d_{i,k,c}\right) + Mq_{i,k,t(\delta)},\ i \in T^{set},\ k \in B^{n,p},\ c = 0,\ n \in N^{set} \tag{16}$$

The (17) indicates that at least one track is to be selected by trains,

$$\sum_{c \in Tr^{n,set}} \left(d_{i,k,c}\right) = 1,\ if\ q_{i,k,t(\delta)} = 0\ and\ i \in T^{set} \tag{17}$$

which can be reformed into (18).

$$\begin{cases} \sum\limits_{c \in Tr^{n,set}} \left(d_{i,k,c}\right) \geq 1 - q_{i,k,t(\delta)}M \\ \sum\limits_{c \in Tr^{n,set}} \left(d_{i,k,c}\right) \leq 1 + q_{i,k,t(\delta)}M \end{cases},\ i \in T^{set},\ k \in B^{n,p},\ n \in N^{set} \tag{18}$$

The (19) indicates that the planned stop in a station cannot be skipped. In China, the passengers take trains with tickets that specifies the planned stations for getting on and off the train. That means the passenger travel plan is made in advance. The (19) is to ensure that the passengers can board the train as planned.

$$D_{i,n} = 1,\;\; i \in T^{set},\; n \in N_{i,d} \tag{19}$$

(5) Departure constraints

If a train departs from a station earlier than the planned time, some passengers may miss the train. Therefore, trains cannot depart from a station earlier than the planned departure times in the origin timetable, see the following inequalities.

$$a_{i,k+1} \geq y_{i,n} - q_{i,k,t(\delta)}M,\; i \in T^{set},\; k \in B^{n,p},\; n \in N_{i,d} \tag{20}$$

(6) Speed grade constraints

The trains must decelerate in advance before stopping at stations, and the trains should accelerate smoothly when departing from stations to guarantee the operation safety. Therefore, the train speed grades in the inbound and outbound section in the station to dwell is constrained to be grade 1, see (21).

$$\beta_{i,k,1} = 1, if\; q_{i,k,t(\delta)} = 0,\; k \in \{B_{n,in} \cup B_{n,out}\},\; D_{i,n} = 1,\; i \in T^{set},\; n \in N^{set} \tag{21}$$

The (21) can be transformed into (22), see

$$\begin{cases} \beta_{i,k,1} \geq 1 - (1 - D_{i,n})M - q_{i,k,t(\delta)}M \\ \beta_{i,k,1} \leq 1 + (1 - D_{i,n})M + q_{i,k,t(\delta)}M \end{cases},\; k \in \{B^{n,in} \cup B^{n,out}\},\; i \in T^{set},\; n \in N^{set} \tag{22}$$

The (23) indicates that at least one speed grade is picked by trains in a block section.

$$\sum\nolimits_{\xi=1}^{E} \beta_{i,k,\xi} = 1,\; i \in T^{set},\; k \in B^{set} \tag{23}$$

The integer variable $g_{i,k}$ denotes the speed grades of train i at block section k. If the speed grade of train i at block section k is grade ξ, i.e., $\beta_{i,k,\xi} = 1$, then we have $g_{i,k} = \xi$ according to (24).

$$g_{i,k} = \sum\nolimits_{\xi=1}^{E} \beta_{i,k,\xi}\xi,\; i \in T^{set},\; k \in B^{set} \tag{24}$$

In consideration of the train acceleration and deceleration characteristics, the difference in speed grades of a train in any two consecutive block sections cannot be bigger than 1, see the following constraints.

$$|g_{i,k+1} - g_{i,k}| \leq 1,\; for\; i \in T^{set},\; k \in B^{set},\; k \neq B \tag{25}$$

To be specific, if the train speed grade at block section k is ξ, the speed grade at the next block section $k+1$ will either increase to $\xi + 1$, decrease to ξ-1, or remain unchanged. The change of speed grades between adjacent block sections is limited, to prevent from sharp train acceleration or deceleration for comfortability. The (25) can be further rewritten into (26).

$$\begin{cases} g_{i,k+1} - g_{i,k} \geq -1 - q_{i,k,t(\delta)}M \\ g_{i,k+1} - g_{i,k} \leq 1 + q_{i,k,t(\delta)}M \end{cases},\; i \in T^{set},\; k \in B^{set},\; k \neq B \tag{26}$$

(7) Constraints related to the temporary speed restrictions

The set of block sections that are covered by the TSRs are denoted by B_{TSR}, the train speed grades in B_{TSR} are limited. The E_δ denotes the maximum allowed speed grades in TSRs. If $q_{i,k,t(\delta)} = 0$ and $t(\delta) < a_{i,k} < t(\delta) + H$ that denotes the train i is to travel at the block section k during the δ-th time horizon, we have $x_{i,k,1} = 1$ according to (27), and we also have $x_{i,k,2} = 1$ according to (28), therefore we have $x_{i,k,1} + x_{i,k,2} = 2$ and the speed grades in B_{TSR} are limited below E_δ according to (29). Otherwise, the train speed grades are not limited.

$$\frac{a_{i,k} - t(\delta)}{M} - q_{i,k,t(\delta)}M \leq x_{i,k,1} \leq \frac{a_{i,k} - t(\delta)}{M} + 1 + q_{i,k,t(\delta)}M,\; i \in T^{set},\; k \in B_{TSR} \tag{27}$$

$$\frac{t(\delta) + H - a_{i,k}}{M} - q_{i,k,t(\delta)}M \leq x_{i,k,2} \leq \frac{t(\delta) + H - a_{i,k}}{M} + 1 + q_{i,k,t(\delta)}M,\; i \in T^{set},\; k \in B_{TSR} \tag{28}$$

$$\sum\nolimits_{\xi=1}^{E_\delta} \beta_{i,k,\xi} \geq x_{i,k,1} + x_{i,k,2} - 1 - q_{i,k,t(\delta)}M,\; i \in T^{set},\; k \in B_{TSR} \tag{29}$$

(8) Headway constraints

If two trains depart from a station consecutively, the minimum departure headway h_d should be respected for safety concerns, see (30). The binary variable $\lambda_{i,j,n}$ denotes the departure sequence of train i and j from station n.

$$\begin{cases} a_{j,k} - a_{i,k} \geq h_d - M\left(1 - \lambda_{i,j,n} + q_{i,k,t(\delta)} + q_{j,k,t(\delta)}\right) \\ a_{i,k} - a_{j,k} \geq h_d - M\left(\lambda_{i,j,n} + q_{i,k,t(\delta)} + q_{j,k,t(\delta)}\right) \end{cases},\; \forall i,\; j \in T^{set},\; i \neq j,\; k \in B^{n,out},\; n \in \{N_{i,d} \cup N_{j,d}\} \tag{30}$$

Similarly, the minimum arrival headway h_a should be respected by any two trains arriving at the same station continuously, see (31). The $\lambda_{i,j,n\text{-}1}$ denotes the departure sequence of train i and j from station n-1, which is also the arrival sequence at the next station n, because overtaking of trains is only allowed at stations.

$$\begin{cases} a_{j,k} - a_{i,k} \geq h_a - M\left(1 - \lambda_{i,j,n-1} + q_{i,k,t(\delta)} + q_{j,k,t(\delta)}\right) \\ a_{i,k} - a_{j,k} \geq h_a - M\left(\lambda_{i,j,n-1} + q_{i,k,t(\delta)} + q_{j,k,t(\delta)}\right) \end{cases}, \ \forall i,\ j \in T^{set},\ i \neq j,\ k \in B^{n,in},\ n \in \{N_{i,d} \cup N_{j,d}\} \tag{31}$$

Moreover, the departure-arrival headway h_{ad} must be respected to avoid the risk of rear-head collisions of trains. For instance, in Fig. 11, train i and train j use the same track (i.e., track 1) to dwell at a station. Train j can only occupy track 1 after the train i leaves it. The departure-arrival headway is the minimum time difference from train i leaving track 1 to train j arriving at track 1. In (32), the binary variable $d_{i,k,c}$ denotes whether train i uses the track c at the block section k. If train i or train j has passed block section k, we have $q_{i,k,t(\delta)} = 1$ or $q_{j,k,t(\delta)} = 1$, and the inequalities become invalid. Otherwise, if train i and j use the same track c, and train i is the former train, we have $d_{i,k,c} = d_{j,k,c} = 1$ and $\lambda_{i,j,n\text{-}1} = 1$, then we have $a_{j,k}\text{-}a_{i,k+1} \geq h_{da}$ according to (32), therefore the departure-arrival headway is respected.

$$\begin{cases} a_{j,k} - a_{i,k+1} \geq h_{da} - M\left(3 - d_{i,k,c} - d_{j,k,c} - \lambda_{i,j,n-1} + q_{i,k,t(\delta)} + q_{j,k,t(\delta)}\right) \\ a_{i,k+1} - a_{j,k} \geq h_{da} - M\left(2 - d_{i,k,c} - d_{j,k,c} + \lambda_{i,j,n-1} + q_{i,k,t(\delta)} + q_{j,k,t(\delta)}\right) \end{cases},$$

$$\forall i,\ j \in T^{set},\ i \neq j,\ k \in B^{n,p},\ n \in N^{set},\ n \neq 1,\ c \in Tr^{n,set} \tag{32}$$

(9) Safety distance constraints

Safety distances between any two consecutive running trains must be guaranteed. If train j travels behind train i, as presented in Fig. 5, the safety distance from train j to train i is decided in real time by the speed of train j. The (33) and (34) are the safety distance constraints that keep enough free sections between any two trains in open-track sections.

$$a_{j,k} - a_{i,v} \geq -\left(2 - \lambda_{i,j,n} - \beta_{j,k,\xi} + q_{i,v,t(\delta)} + q_{j,k,t(\delta)}\right)M$$

$$\forall i,\ j \in T^{set},\ i \neq j,\ \forall k,\ v \in B_{<n,n+1>},\ v = k + \xi + 1,\ n \in N^{set},\ n \neq N \tag{33}$$

$$a_{i,k} - a_{j,v} \geq -\left(1 + \lambda_{i,j,n} - \beta_{i,k,\xi} + q_{i,k,t(\delta)} + q_{j,v,t(\delta)}\right)M$$

$$\forall i,\ j \in T,\ i \neq j,\ \forall k,\ v \in B_{<n,n+1>},\ v = k + \xi + 1,\ n \in N^{set},\ n \neq N \tag{34}$$

(10) Objective functions

The $a_{i,B}$ denotes the arrival time of train i at the last block section. The aim of (35) is to make trains arrive at the terminal station as soon as passible, therefore, the total train traveling times are minimized.

$$F_1 = \sum_{i \in T} a_{i,B} \tag{35}$$

The objective (36) is to minimize the number of times the train speed grades change. Therefore, the operation of trains is smoothed.

$$F_2 = \sum_{k=1}^{B-1} \mu_{i,k} \tag{36}$$

The variable $\mu_{i,k}$ is constrained by (37).

$$\mu_{i,k} = \begin{cases} 1,\ if\ g_{i,k} \neq g_{i,k+1}\ and\ q_{i,k,t(\delta)} = 0, \\ 0,\ otherwise. \end{cases} \tag{37}$$

In this paper, we transform (37) into (38). The two constraints are different to some extent. However, by minimizing (36) the two constraints are equivalent. When $q_{i,k,t(\delta)} = 0$ is true, if $g_{i,k+1} = g_{i,k}$, we have $\mu_{i,k} \geq 0$ according to (38), eventually we have $\mu_{i,k} = 0$ by minimizing (36), and if $g_{i,k+1} \neq g_{i,k}$, we have $|g_{i,k+1}\text{ - }g_{i,k}| = 1$ due to (26), therefore we have $\mu_{i,k} \geq 1$, and eventually we have $\mu_{i,k} = 1$ by minimizing (36).

$$\begin{cases} \mu_{i,k} \geqslant g_{i,k+1} - g_{i,k} - M \cdot q_{i,k,t(\delta)} \\ \mu_{i,k} \geqslant g_{i,k} - g_{i,k+1} - M \cdot q_{i,k,t(\delta)} \end{cases} \tag{38}$$

Finally, the objective function is formulated as follows, and Q_1 and Q_2 are the weights.

$$\text{min}imize\ (Q_1 \cdot F_1 + Q_2 \cdot F_2) \tag{39}$$

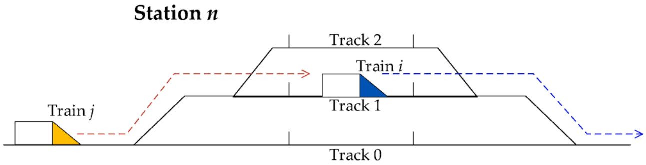


Fig. 11. Illustration of the departure-arrival headway.

6. Case studies

Case studies are carried out based on real-world data from the Beijing-Tianjin intercity railway line [37]. Fig. 12 presents the sketch map. The MILP model is solved in a framework of the rolling horizon algorithm, and the commercial solver IBM ILOG CPLEX 12.9.0 is applied to solve the model in each time horizon to optimality (with a gap of 0). The experiments are all executed on a machine with an Intel(R) Core (TM) i5-10300 processor CPU 2.50 GHz and 16.00 GB memory.

Headway times are all set as $h_a = h_d = h_{ad} = 120$ s, maximum dwelling time D^{max} and minimum dwelling time D^{min} are set as $D^{max} = 20$ min and $D^{min} = 3$ min, the rolling horizon parameters are $r = 10$ min, and $H = 20$ min, the weights are set as $Q_1 = 1$ and $Q_2 = 1$. There are 3 parallel tracks in each intermediate station in the downstream direction, see Fig. 3.

6.1. Case study 1

Fig. 13 presents the origin timetable between 6:01am and 11:15am, which contains the trajectories of 30 trains. The time 6:01 am is set as zero in the horizontal axis direction. The colors of train trajectories denote the train speed grades in each block section. We can see that train 1 travels from Beijing to Tianjin with speed grade 4 mostly. After 2 min of stopping at Wuqing, train 1 accelerates to speed grades 6 and arrives at Tianjin eventually. Train 2, train 3, and train 4 travel directly from Beijing to Tianjin with speed grades 6 mostly.

In this case study, 15 block sections in the Yongle-Wuqing segment are influenced by TSRs. The fluctuation of the speed limit value is illustrated in Fig. 14. The speed restriction happens at time $\widetilde{T}_1$ and the value is $\widetilde{V}_1$ km/h, then changes to $\widetilde{V}_2$ at time $\widetilde{T}_2$, and eventually ends at time $\widetilde{T}_3$. The time $\widetilde{T}_1$, $\widetilde{T}_2$ and $\widetilde{T}_3$, and the speed limits $\widetilde{V}_1$ and $\widetilde{V}_2$ are all random parameters. We assume that these random variables accord with the uniform distribution, and the boundaries are given in Fig. 14. The values of the random variables are realized in simulations.

Fig. 15(a) presents the timetable calculated in the 1-th time horizon. At time $t(1)$, TSRs are detected and a new timetable is generated for the next H minutes. The portion marked by the dotted rectangle denotes the range affected by the TSRs in time and space in each time horizon.

We can see that train speed grades are changed compared to the origin timetable. In the 1-th time horizon, train speed is limited to grade 1 in the affected area. Train 1 decelerates to grade 1 before entering the affected area. To see the change of colors of the trajectory in the locally enlarged part in Fig. 15(a), we learn that the train speed grades reduce from grade 6 to grade 1 step by step, therefore the smooth deceleration of train 1 can be guaranteed. Train 2, train 3, and train 4 do not travel in the affected area in the 1-th time horizon.

Fig. 15(b) presents the train timetable calculated in the 2-th time horizon that begins at time $t(2)$. The speed restriction detected at time $t(2)$ is different from that detected at $t(1)$, and the maximum allowed speed grade in the affected area changes to grade 4. The locally enlarged part in Fig. 15(b) shows that the speed grade of train 1 changes to grade 2 in the affected area, which is different from the trajectory in the 1-th time horizon. The speed grades of train 2 and train 3 are reduced to grade 4 in the affected area. By comparing the colors of the train trajectories in the 1-th and 2-th time horizons, we can see that the proposed approach dynamically adjusts the train speed according to the TSRs newly detected, and the train speed grades change step by step for smooth traveling of trains.

Fig. 16 presents the timetable calculated in the 3-th and 4-th time horizons, the maximum train speed grades are limited to grade 4

Fig. 12. Sketch map of Beijing-Tianjin intercity railway line.

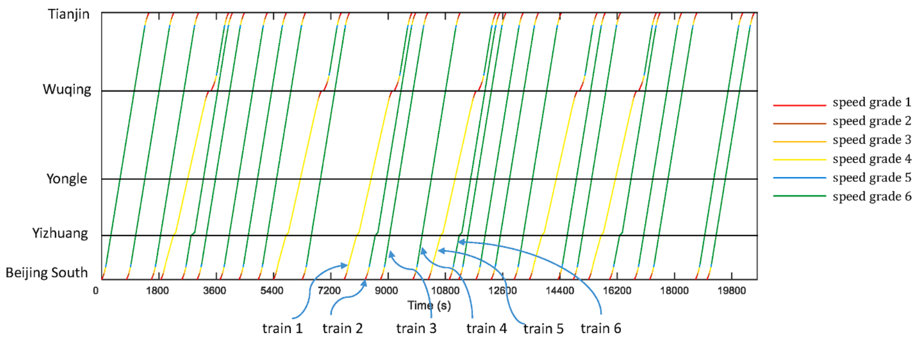


Fig. 13. The origin train timetable.

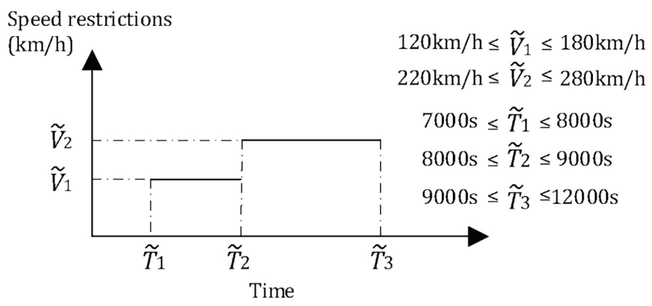


Fig. 14. The fluctuation of the TSRs in case study 1.

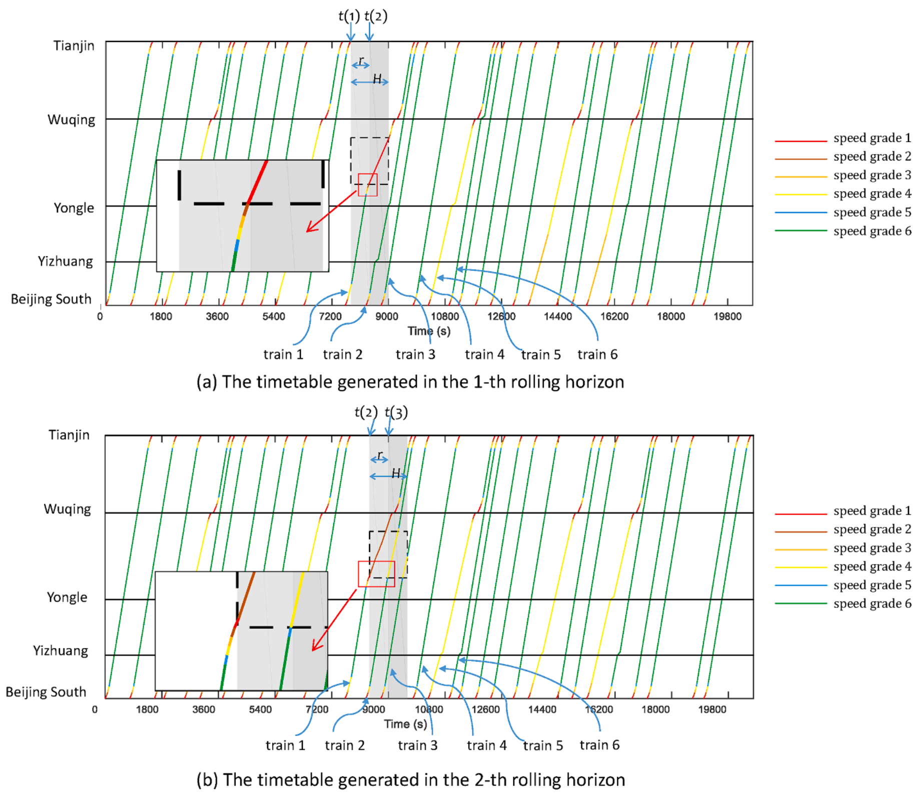
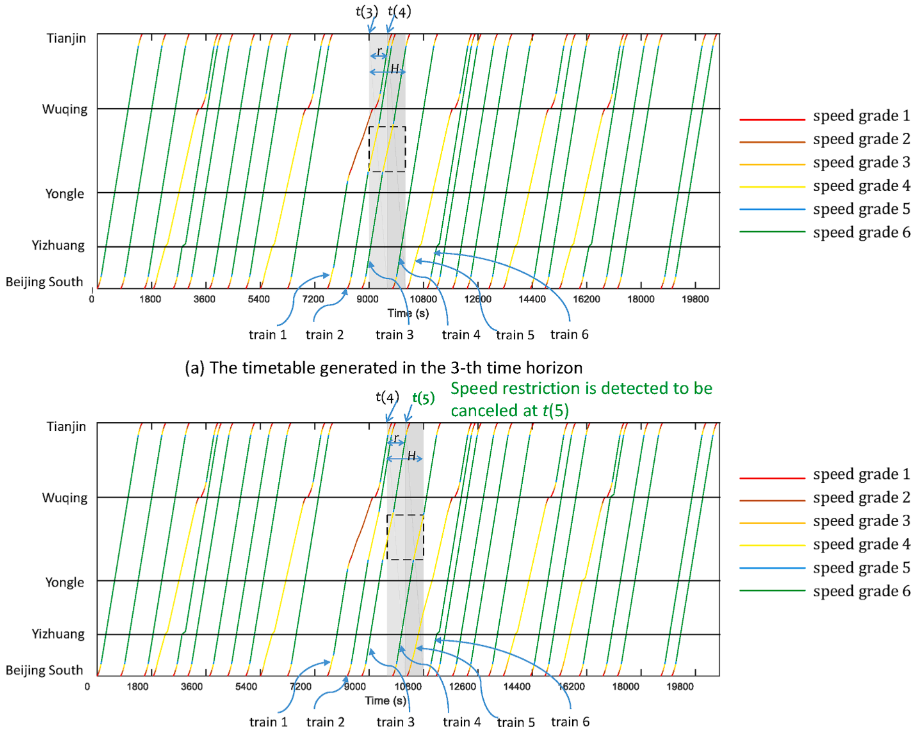


Fig. 15. Train timetable generated in the 1-th and 2-th time horizons in case study 1.



(a) The timetable generated in the 3-th time horizon

(b) The timetable generated in the 4-th time horizon

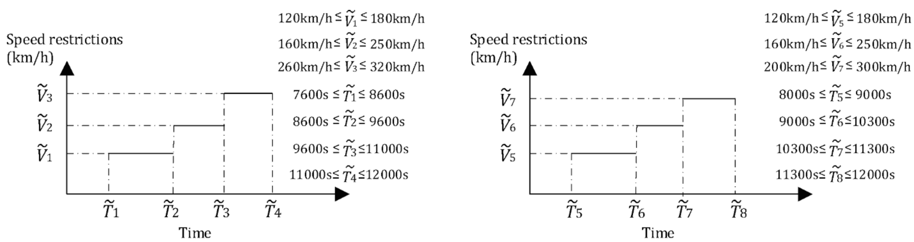
Fig. 16. Train timetable generated in the 3-th and 4-th time horizons in case study 1.

in the affected area. The speed grades of train 4 are not affected till the 4-th time horizon. At time $t(5)$ which is the beginning of the 5-th time horizon, speed restriction is detected to retreat. Therefore, the algorithm ends timely at the end of the 4-th rolling horizon, after which the train speeds are no longer affected.

6.2. Case study 2

Different from case study 1 which assumes only a single area is affected by TSRs, in case study 2 multiple areas are affected. The origin timetable and the parameters are the same as in case study 1. The change of TSRs is depicted in Fig. 17. In the Yongle-Wuqing segment, 15 block sections are affected, the fluctuation of speed restriction in this segment is depicted in Fig. 17(a), and the speed restriction happens at time $\widetilde{T}_1$ with value of $\widetilde{V}_1$, and changes at time $\widetilde{T}_2$ to $\widetilde{V}_2$, then changes to $\widetilde{V}_3$ at time $\widetilde{T}_3$, eventually ends at time $\widetilde{T}_4$. The fluctuation of speed restriction in the Wuqing-Tianjin segment is illustrated in Fig. 17 (b), where the speed restriction affects 10 block sections, it happens at time $\widetilde{T}_5$ with value of $\widetilde{V}_5$, and changes to $\widetilde{V}_6$ at time $\widetilde{T}_6$, then changes to $\widetilde{V}_7$ at time $\widetilde{T}_7$, eventually ends at time $\widetilde{T}_8$. The speeds $\widetilde{V}_1$, $\widetilde{V}_2$, $\widetilde{V}_3$, $\widetilde{V}_5$, $\widetilde{V}_6$ and $\widetilde{V}_7$ and times $\widetilde{T}_1$, $\widetilde{T}_2$, $\widetilde{T}_3$, $\widetilde{T}_4$, $\widetilde{T}_5$, $\widetilde{T}_6$, $\widetilde{T}_7$ and $\widetilde{T}_8$ are all random variables that accord with uniform distribution, and the boundaries are also given in Fig. 17.

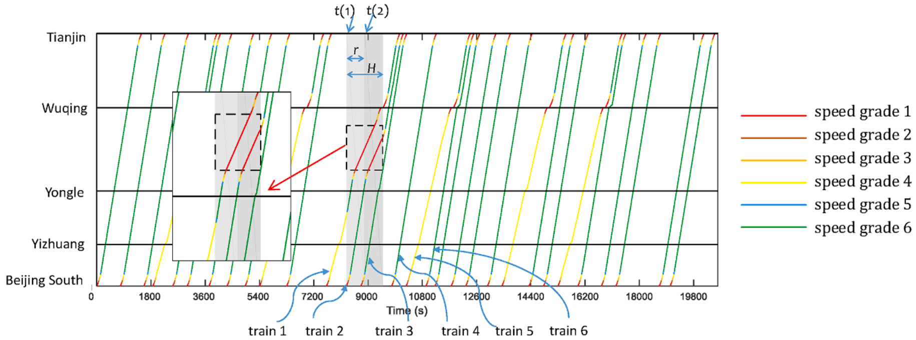
Fig. 18 presents the timetables generated in the 1-th and 2-th time horizons. We can see that at time $t(1)$ the speed restriction in the Yongle-Wuqing segment is first detected and the 1-th time horizon starts, limiting the train speed grades to grade 1. We compare the



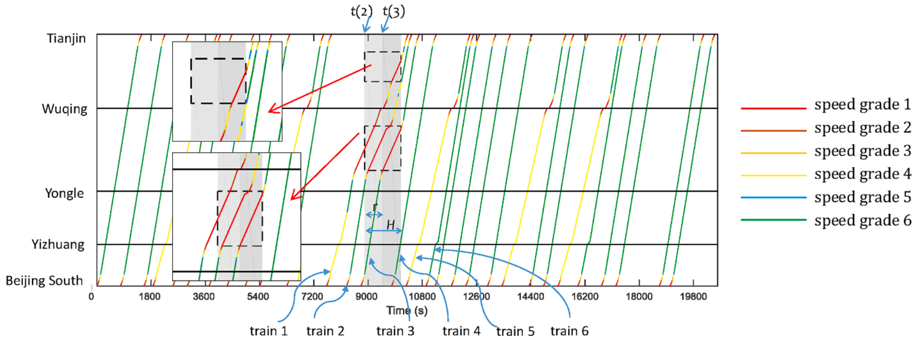
(a) Speed restriction on the Yongle-Wuqing segment

(b) Speed restriction on the Wuqing-Tianjin segment

Fig. 17. The fluctuation of the TSRs in case study 2.

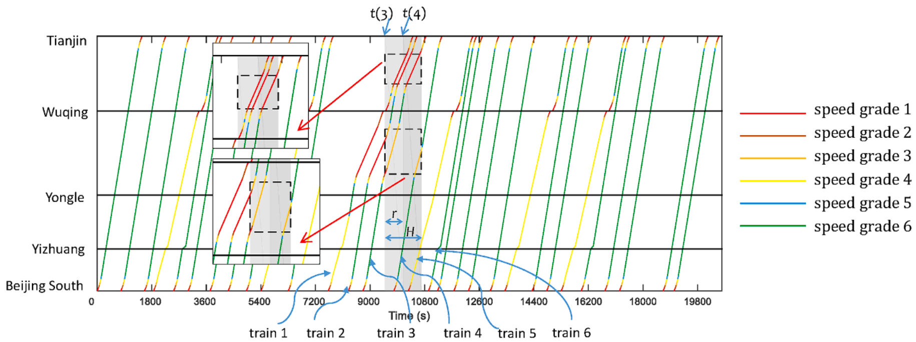


(a) The timetable generated in the 1-th time horizon

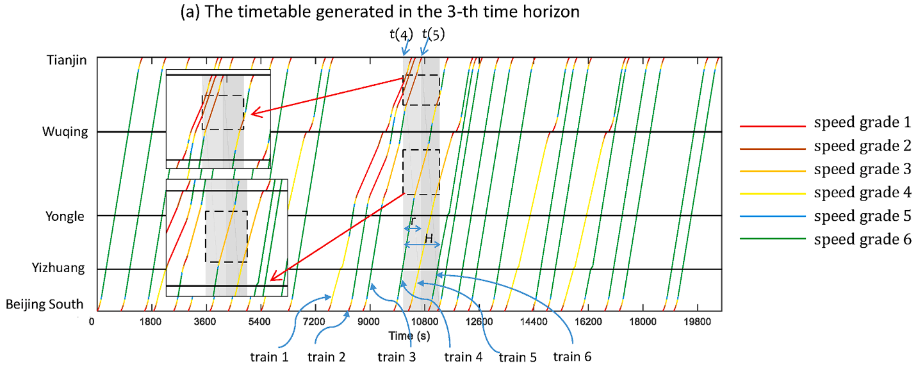


(b) The timetable generated in the 2-th time horizon

Fig. 18. Train timetable generated in the 1-th and 2-th time horizons in case study 2.



(a) The timetable generated in the 3-th time horizon



(b) The timetable generated in the 4-th time horizon

Fig. 19. Train timetable generated in the 3-th and 4-th time horizons in case study 2.

trajectory of train 1 in Fig. 18(a) and Fig. 13. In the origin timetable, train 1 travels from Beijing to Tianjin with speed grade 4 mostly, and the speed grade 5 and 6 are never used before stopping at Wuqing station. However, train 1 starts to accelerates to speed grade 6 at time $t(1)$ before the affected area, see Fig. 18(a). The reason is that by accelerating to higher speed in the normal area, train delays caused by TSRs can be reduced. Therefore, we can learn that the proposed approach can alleviate train delays by managing train speeds.

The 2-th time horizon starts at time $t(2)$, both in the Yongle-Wuqing and Wuqing-Tianjin segments the TSRs are detected. In Fig. 18 (b), train 3 accelerates to speed grade 6 as soon as possible after leaving the affected area in the Yongle-Wuqing segment, but train 2 does not. The reason is that if train 2 accelerates like train 3, train 2 will suffer the speed restriction in the Wuqing-Tianjin segment. Therefore, to avoid entering the speed restriction area again, train 2 accelerates more slowly, and for train 2 delay is also optimized. Therefore, we can see that the proposed approach can reasonably manage the train speeds according to the TSRs in multiple areas to reduce the train delay.

Fig. 19 presents the timetable calculated in the 3-th and 4-th time horizons. The speed restriction in the Yongle-Wuqing segment changes in the 3-th time horizon, limiting the train speed grades to grade 3. By comparing the colors of the trajectories of train 2 and train 3 in Fig. 18 and Fig. 19, we can see that the train speed grades in the affected area in the Yongle-Wuqing segment dynamically change from grade 1 to grade 3. To update the train speed grades timely, train delays can be further reduced. The speed restriction in the Wuqing-Tianjin segment changes in the 4-th time horizon and limits the train speed to grade 2, and the speed grades of train 2, train 3 and train 4 in the affected area also change dynamically.

By comparing the colors of trajectories in the affected area in Fig. 19(b) and Fig. 20(a), we can see that the TSRs change both in the two affected areas at $t(5)$. At time $t(5)$, in the affected area in the Yongle-Wuqing segment, train speed grades are limited to grade 5, while in the Wuqing-Tianjin segment train speed grades are limited to grade 4.

In Fig. 20(a), the speed grades of train 4 and train 6 change to grade 5 in the affected area in the Yongle-Wuqing segment, while the speed grade of train 5 remains to grade 4. The reason is that train 5 travels at speed grade 4 originally in this area, see the origin timetable in Fig. 13, therefore changing the speed grade of train 5 is unnecessary. At time $t(7)$, the TSRs are detected to be canceled, therefore the algorithm ends naturally at the end of the 6-th time horizon.

7. Conclusion

This paper focused on dynamically rescheduling trains with speed management in uncertain disruptions. The disruption mainly considered is the temporary speed restrictions. The uncertainties of TSRs are well handled by applying the rolling horizon algorithm. We first formulated a mixed-integer linear programming model, two kinds of 0–1 parameters were innovatively designed to denote train position, and some integer parameters were used to denote train speed grades in block sections. The rolling horizon approach was

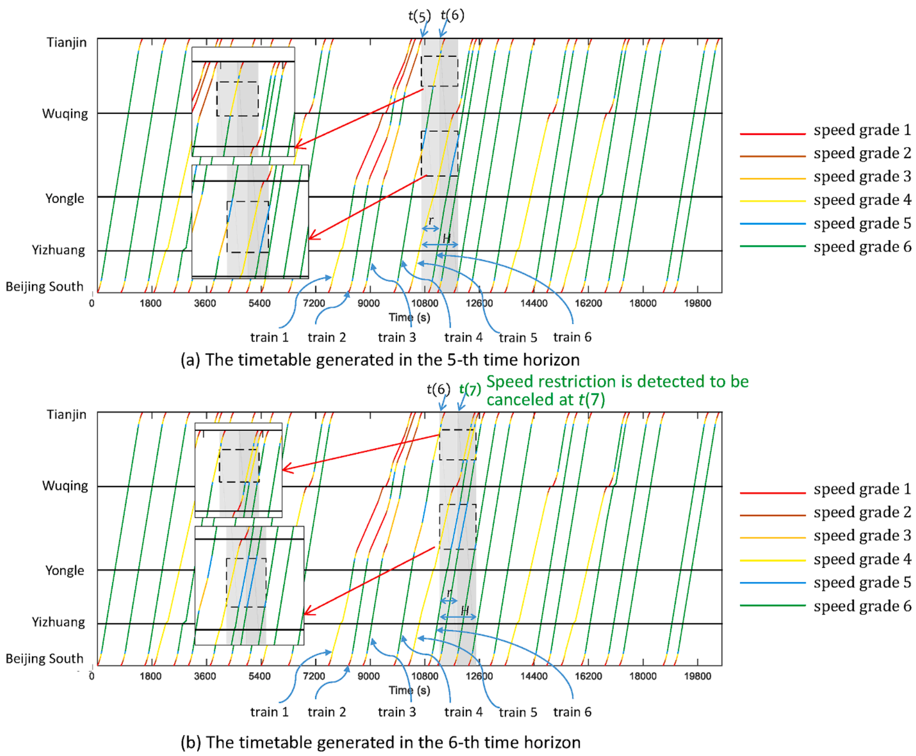


Fig. 20. Train timetable generated in the 5-th and 6-th time horizons in case study 2.

used to solve the proposed model repeatedly based on the newest information detected, e.g., train speeds, train positions and speed restrictions. Therefore, the train speeds and arrival times for each block section can be dynamically adjusted to minimize train delay. The train speed grades change step by step to smooth the train trips for comfortability. The two case studies were based on the single-area TSRs and the multiple-area TSRs, respectively. Results verified that the proposed approach effectively handles the uncertainties of TSRs by timely and reasonably managing speeds for each train, while the smooth traveling, and the punctuality of trains are all optimized.

For future work, other kinds of disruptions or disturbances such as railway blockage, and emergency train deceleration can be considered in the model. To further calculate the train speed profiles for saving energy is also important in future work because the train automatic operation system drive trains based on the predesigned speed profiles. Moreover, railways lines in China are mostly connected, forming railway networks that connect major cities. Managing trains from the perspective of the whole railway network is important in future work. Finally, the passenger demand is to be considered in the formulation of optimization models in future work.

CRediT authorship contribution statement

Sairong Peng: Methodology, Data curation, Writing – original draft. **Xin Yang:** Conceptualization, Methodology, Writing – original draft. **Shuxin Ding:** Methodology, Writing – review & editing. **Jianjun Wu:** Conceptualization, Supervision, Funding acquisition. **Huijun Sun:** Investigation, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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