High-Speed Railway Train Timetable Rescheduling in Case of a Stochastic Section Blockage

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***Abstract*—In this paper, the uncertain duration of the section blockade is considered for the high-speed railway train timetable rescheduling problem with the entire section blockade. The objective is to minimize the conditional value-at-risk of the train delay by retiming, reordering, etc. A two-stage stochastic mixed-integer linear programming model is established for the train rescheduling problem in a macroscopic view. Some model approximation methods with scenarios are applied to speed up the solution process. The proposed models are solved by the GUROBI commercial solver. Numerical experiments are performed based on the Beijing-Tianjin intercity railway to show the effectiveness of the proposed stochastic programming model. The experimental results showed that the proposed model with scenario reduction and order scenario reduction could get a high-quality solution with a shorter computation time.**

***Index Terms*—High-speed railway, train timetable rescheduling, mixed-integer linear programming, stochastic programming, uncertainty, conditional value-at-risk**

I. Introduction

High-speed railway (HSR) train dispatching and commanding is the central part of high-speed train operation. Train timetable rescheduling (TTR) is conducted when predefined train operations are affected by inevitable emergencies, e.g., train failure, natural disasters, etc. Disturbances are small perturbations of the railway system which can be handled by adjusting the timetable. For large incidents, which are defined as disruptions, timetable, rolling stock, and crew should be modified [1]. There are many uncertain features in actual train operations, which account for an efficient TTR method for increasing the train operation efficiency.

Most of the studies consider the parameters for the emergencies are deterministic [2], [3]. For studies with uncertain parameters, Yang *et al.* [4] proposed a two-stage fuzzy programming problem to deal with the uncertain TTR problem. The recovery time was regarded as a fuzzy uncertain variable, and the expectation of the train delay time was minimized. Li *et al.* [5] considered the stochastic recovery time, minimizing the delay cost and the expected track changing cost with track-backup. Meng and Zhou [6] proposed a two-stage stochastic programming model with a stochastic section running time and capacity loss duration in a single-track. The objective was to minimize the arrival time deviation in the final station. The problem was solved in a scenario-based rolling horizon framework with branch-and-bound. Yang *et al.* [7] improved [4] in a railway network, minimizing the total fuzzy delays under confidence level α with a two-stage 0-1 integer fuzzy

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programming model. Zhu and Goverde [8] proposed a rolling horizon two-stage stochastic timetable rescheduling model to deal with stochastic disruptions, considering retiming, reordering, canceling, adding stops, and flexible short-turning. Hong *et al.* [9] proposed a mixed-integer linear programming (MILP) model with large stochastic disruptions to model the robust capacitated train rescheduling problem with passenger reassignment and solved it by a two-stage approach.

Section blockage is a typical disruption, a variety of studies have analyzed the TTR problem under this condition. Most of the studies consider that the duration of the blockage is deterministic. However, the actual value is hard to obtain. Meanwhile, although some studies consider uncertainty, the risk is omitted. This paper proposes a scenario-based approach to model the stochastic disruptions as a stochastic programming model with conditional value-at-risk (CVaR) [10]. Linearization, model reformulation, and approximation methods are developed to decrease the complexity of the model. Simulation results have demonstrated the effectiveness of the proposed model, which provides a new approach for TTR problems under uncertainty.

We summarize our contributions as follows:

1) We present a CVaR-based TTR model under uncertainty with stochastic disruption scenarios.
2) Model transformation and approximation methods are applied to speed up the computation and provide an efficient upper bound.

The rest of this paper is organized as follows. In Section II, we describe the CVaR-based TTR problem. The proposed model is evaluated by experiments in Section III. Finally, we conclude our work in Section IV.

II. Problem Formulation

This section introduces a mixed-integer nonlinear programming (MINLP) model to formulate the TTR problem with stochastic section blockage. This model minimizes the CVaR of the arrival and departure cost with operational constraints.

A. Assumptions

1) Rolling stock rescheduling and crew rescheduling are not considered.
2) The upstream and downstream trains are operated separately on their side of tracks and platforms. We only need to reschedule on one side.
3) Retiming and reordering are considered, whereas no trains are canceled in TTR.
4) All the train should follow their original schedules before disruption happens.
5) Disruption considered is a complete section blockage between two adjacent stations.
6) There is only one disruption, whose duration is a stochastic variable with known distribution.
7) For trains already entered the disruption section are assumed to have passed the blockage in the section.
8) Trains are allowed to arrive early at the station, but they should not depart early.

TABLE I
Summary of notations.

Symbol	Description
	Indices
$i, l \in I$	Index of train
$j \in J$	Index of station
$k \in K$	Index of section
$s \in S$	Index of scenario
	Parameters
I	Set of trains
J	Set of stations
K	Set of sections
c_{ij}^s	Unit cost when train i arrives early at station j
e_{ij}^s	Unit cost when train i arrives late at station j
e_{ij}^e	Unit cost when train i leaves late at station j
o_{ij}^s	The start time of train i at station j in the original schedule
o_{ij}^e	The end time of train i at station j in the original schedule
Y_{ij}	The train stop indicator in the original schedule, 1 if train i stops at station j; 0 otherwise
α_i	The departure station for train i
β_i	The terminal station for train i
d_{ij}	The minimum dwell time at station j for train i
r_k^{min}	The minimum running time at section k
r_k^s	The additional time caused by starting at section k
r_k^e	The additional time caused by stopping at section k
H_k	The minimum headway between two consecutive trains of the same direction at section k
M	A large positive number
S	Set of scenarios
H_{dis}^s	Start time of the disruption
$T_{dis}(s)$	Duration of the disruption for scenario s
k^*	Index of the disrupted section
	Decision variables
$x_{ij}^s(s)$	Start time for train i at station j for scenario s
$x_{ij}^e(s)$	End time for train i at station j for scenario s
$q_{ilk}(s)$	Traversing order for scenario s, 1 if train i traverses at section k before train l; 0 otherwise
$y_{ij}(s)$	Actual train stop indicator for scenario s, 1 if train i stops at station j; 0 otherwise

9) The capacity of the station is unlimited.

B. Parameters and Decision Variables

Table I summarizes all the notations that are used throughout this paper.

C. Objective Function

The total arrival and departure cost for all trains under scenario s is defined as:

$$D(\mathbf{x}, \mathbf{q}, \mathbf{y}, s) = \sum_{i \in I} \sum_{j=\alpha_i}^{\beta_i} c_{ij}^s [o_{ij}^s(s) - x_{ij}^s(s)]^+ + e_{ij}^s [x_{ij}^s(s) - o_{ij}^s(s)]^+ + e_{ij}^e (x_{ij}^e(s) - o_{ij}^e) \quad (1)$$

where I denotes the set of trains, $[t]^+ = max\{0, t\}$. $\mathbf{x} = [x_{ij}^s(s), x_{ij}^e(s)]_{|I| \times |J| \times |S|}$, $\mathbf{q} = [q_{ilk}(s)]_{|I| \times |I| \times |K| \times |S|}$ and $\mathbf{y} = [y_{ij}(s)]_{|I| \times |J| \times |S|}$ are the decision variables.

For S scenarios, the corresponding CVaR value is calculated by [10]:

$$\mathrm{CVaR}_\beta(D(\mathbf{x}, \mathbf{q}, \mathbf{y}, s)) =$$

$$\min_{\alpha \in \mathbb{R}} \left\{ \alpha + \frac{1}{1-\beta} \sum_{s \in S} p_s \left[D(\mathbf{x}, \mathbf{q}, \mathbf{y}, s) - \alpha \right]^+ \right\} \quad (2)$$

where p_s is the probability of scenario s, $\alpha \in \mathbb{R}$ denotes an auxiliary variable, which is the value-at-risk (VaR) after optimization. The confidence level of CVaR is described as $\beta \in [0, 1]$, which measures the risk preference of the decision-maker (DM). The confidence level β determines the number of scenarios used for calculation. S denotes the set of scenarios.

Remark 1: When $\beta = 0$, the DM is risk-neutral, and the CVaR operator is equivalent to the expectation operator. When $\beta = 1$, the DM is risk-averse, and the CVaR operator is equivalent to the maximum operator. Only the scenario with the maximum $D(\mathbf{x}, \mathbf{q}, \mathbf{y}, s)$ is used for calculation.

D. Constraints

Several constraints for train operations are described as follows.

1) Dwell Time Constraints: The dwell time at station j is larger than the minimal dwell time interval to ensure the operations at the station, e.g., passenger unloading and loading, crew rescheduling, etc.

$$x^e_{ij}(s) - x^s_{ij}(s) \geq d_{ij} \ \forall i \in I; j \in \{\alpha_i, \ldots, \beta_i\}; s \in S \quad (3)$$

2) Running Time Constraints: The running time in section k should be greater than the minimum running time.

$$x^s_{i,j+1}(s) - x^e_{ij}(s) \geq r^{min}_k + r^s_k y_{ij}(s) + r^e_k y_{i,j+1}(s) \quad (4)$$

for all $i \in I$, $j \in \{\alpha_i, \ldots, \beta_i - 1\}$, $k = j$, $s \in S$.

3) Headway Constraints: The headway between two adjacent trains should be greater than the minimum headway to ensure safety. The trains are assumed with constant speed at sections. Therefore, we need to ensure that the headway is satisfied at the start time and end time in stations.

$$x^e_{lj}(s) - x^e_{ij}(s) \geq H_k q_{ilk}(s) - M(1 - q_{ilk}(s)) \quad (5)$$
$$x^s_{l,j+1}(s) - x^s_{i,j+1}(s) \geq H_k q_{ilk}(s) - M(1 - q_{ilk}(s)) \quad (6)$$

for all $i, l \in I$, $i \neq l$, $j \in \{\alpha_i \vee \alpha_l, \ldots, \beta_i \wedge \beta_l - 1\}$, $k = j$, $s \in S$.

4) Close-to-Favorite-Schedule Constraints: All the trains should follow their favorite timetable before the disruption happens.

$$x^s_{ij}(s) = o^s_{ij} \ \forall i \in I; j \in \{\alpha_i, \ldots, \beta_i\}; s \in S : o^s_{ij} \leq H^s_{dis} \quad (7)$$
$$x^e_{ij}(s) = o^e_{ij} \ \forall i \in I; j \in \{\alpha_i, \ldots, \beta_i\}; s \in S : o^s_{ij} \leq H^s_{dis} \quad (8)$$

5) Initial Rescheduling Time Constraints: The affected trains should leave the station before the disrupted section after the disruption ends.

$$x^e_{ik^*}(s) \geq H^s_{dis} + T_{dis}(s) \quad (9)$$

for all $i \in I$, $s \in S$ where $H^s_{dis} \leq o^e_{ij} \leq H^s_{dis} + T_{dis}(s)$.

6) Arrival Time Constraints: Trains are not allowed to depart earlier than the originally scheduled time to prevent passengers from missing trains.

$$x^e_{ij}(s) \geq o^e_{ij} \ \forall i \in I; j \in \{\alpha_i, \ldots, \beta_i\}; s \in S \quad (10)$$

7) Traversing Order Constraints: For two adjacent trains, either one can traverse at a section before the other.

$$q_{ilk}(s) + q_{lik}(s) = 1 \quad (11)$$

for all $i, l \in I$, $i \neq l$, $j \in \{\alpha_i \vee \alpha_l, \ldots, \beta_i \wedge \beta_l - 1\}$, $s \in S$.

$$q_{ilk^*}(s = 1) = \cdots = q_{ilk^*}(s = S) \ \forall i, l \in I; i \neq l \quad (12)$$

where $q_{ilk^*}(s)$ denotes the traversing order for trains at the disruption section, which is a first-stage decision variable. It means the value remains the same under different scenarios, which is a *here-and-now* type problem, as shown in (12). In the second stage, the rescheduled arrival, departure time, traversing order in other sections, and train stop indicator are decided under each realized disruption time in the scenario. It is a *wait-and-see* type problem, where decisions are made when the random quantities can be observed.

8) Train Stop Constraints: Adding stops is allowed for trains, whereas canceling stops is not allowed.

$$y_{ij}(s) \leq x^e_{ij}(s) - x^s_{ij}(s) \ \forall i \in I; j \in \{\alpha_i, \ldots, \beta_i\}; s \in S \quad (13)$$
$$y_{ij}(s) \geq \frac{x^e_{ij}(s) - x^s_{ij}(s)}{M} \ \forall i \in I; j \in \{\alpha_i, \ldots, \beta_i\}; s \in S \quad (14)$$
$$y_{ij}(s) \geq Y_{ij} \ \forall i \in I; j \in \{\alpha_i + 1, \ldots, \beta_i - 1\}; s \in S \quad (15)$$
$$y_{ij}(s) = Y_{ij} \ \forall i \in I; j \in \{\alpha_i, \beta_i\}; s \in S \quad (16)$$

9) Decision Variable Constraints: The following constraints define the domain of the decision variables:

$$x^s_{ij}(s), x^e_{ij}(s) \geq 0 \ \forall i \in I; j \in \{\alpha_i, \ldots, \beta_i\}; s \in S \quad (17)$$
$$q_{ilk}(s) \in \{0, 1\} \ \forall i, l \in I; i \neq l; j \in \{\alpha_i \vee \alpha_l, \ldots, \beta_i \wedge \beta_l - 1\}; k = j; s \in S \quad (18)$$
$$y_{ij}(s) \in \{0, 1\} \ \forall i \in I; j \in \{\alpha_i, \ldots, \beta_i\}; s \in S \quad (19)$$

E. CVaR-based TTR model

The CVaR-based TTR model is formulated to minimize the CVaR of the rescheduling cost (2) under several constraints, that is:

$$\begin{cases} \min \ \text{CVaR}_\beta(D(\mathbf{x}, \mathbf{q}, \mathbf{y}, s)) \\ \text{s.t. Constraints } (3) - (19). \end{cases} \quad (20)$$

Since there are continuous real variables $\mathbf{x}$, 0-1 variables $(\mathbf{q}, \mathbf{y})$, and nonlinear terms (minimax function) in (2), the proposed CVaR-based TTR model (20) belongs to MINLP.

F. Model Reformulation

Linearization should be applied to deal with the proposed MINLP. Besides, several model reformulations are proposed, including scenario reduction and order scenario reduction, which can efficiently deal with the model.

1) Linearization: Due to the nonlinear terms (minimax function) in (2), linearization method is developed. Three auxiliary variables are introduced, i.e., $\mathbf{f}_1 = [f_1^{ij}(s)]_{|I|\times|J|\times|S|}$, $\mathbf{f}_2 = [f_2^{ij}(s)]_{|I|\times|J|\times|S|}$, and $\mathbf{f}_3 = [f_3(s)]_{1\times|S|}$, which are defined as follows:

$$\begin{cases} f_1^{ij}(s) = o_{ij}^s - x_{ij}^s(s) \\ f_2^{ij}(s) = x_{ij}^s(s) - o_{ij}^s \qquad \forall i \in I; j \in \{\alpha_i, \ldots, \beta_i\}; s \in S \\ f_3(s) = D(\mathbf{x}, \mathbf{q}, \mathbf{y}, s) - \alpha \end{cases} \tag{21}$$

Substituting (1) and (2) by (21), a reformulated MILP model is obtained. The CVaR-based TTR model (CVaR-TTR) can be reformulated as follows:

$$\min \ \alpha + \frac{1}{1-\beta} \sum_{s\in S} p_s f^3(s) \tag{22}$$

$$\text{s.t. } f_1^{ij}(s) \geq o_{ij}^s - x_{ij}^s(s) \tag{23}$$

$$f_2^{ij}(s) \geq x_{ij}^s(s) - o_{ij}^s \tag{24}$$

$$f_3(s) \geq \sum_{i\in I} \sum_{j=\alpha_i}^{\beta_i} \Big(c_{ij}^s f_1^{ij}(s) + e_{ij}^s f_2^{ij}(s) + e_{ij}^e (x_{ij}^e(s) - o_{ij}^e) \Big) - \alpha \tag{25}$$

$$f_1^{ij}(s), f_2^{ij}(s), f_3(s), \alpha \geq 0 \tag{26}$$

$$\text{Constraints (3)} - (19). \tag{27}$$

for all $i \in I$, $j \in \{\alpha_i, \ldots, \beta_i\}$, $s \in S$.

2) Scenario Reduction: According *Remark 1*, all the scenarios are used for calculation when $\beta = 0$. When β is greater than 0, the corresponding scenarios related with β are used for calculation, rather than all the scenarios. Therefore, a scenario reduction strategy is proposed to speed up the model according to this problem-specific knowledge. A reformulated model called the CVaR-based TTR model with scenario reduction (CVaR-TTR-SR) is proposed.

If the probability p_s varies under different scenario s, the following lemmas can be used to obtain the reduced scenarios.

Lemma 1: For CVaR-TTR, the searching space decreases if the duration of the disruption increases with a given train traversing order in the disrupted section.

Proof: With a longer duration of the disruption, there will be less possible arrival and departure time for the rescheduling of disrupted trains. ■

Lemma 2: For CVaR-TTR, the total arrival and departure cost for scenario s will have an increasing tendency if the duration of the disruption for scenario s increases with a given train traversing order in the disrupted section.

Proof: Based on *Lemma 1*, since the searching space decreases with the increase in the duration of the disruption, some optimal solutions may violate the constraints, which lead to the increase of the objective value of CVaR-TTR. ■

Lemma 3: For $T_{dis}(s=1) \leq \cdots \leq T_{dis}(s=S)$, the cumulative probability $p_c(s)$ for each scenario is calculated. Then, the number of scenarios is reduced to S_{reduce}, which is calculated by the number of scenarios with cumulative probability greater than the confidence level β, $S_{reduce} = \sum_{s\in S} \operatorname{sgn}(p_c(s) - \beta)$. The reduced scenarios are those with least duration of the disrupiton.

Proof: According to the definition of CVaR, which is calculated by averaging the value for scenarios greater than the value with confidence level β. The scenarios with a cumulative probability $p_c(s)$ greater than β are remained. ■

Lemma 4: If the probability for different scenario s equals to $1/S$, the number of scenarios is reduced to $S_{reduce} = \lceil S \cdot (1-\beta) \rceil$. The remaining scenarios are the top S_{reduce} scenarios with the longest duration of the disruption.

Proof: *Lemma 4* is a special condition of *Lemma 3* with the same p_s. If $n/S > \beta$ and $(n-1)/S < \beta$ ($n \in \{1, \ldots S\}$), then $S_{reduce} = S - n + 1 < S - S\cdot\beta + 1 = S\cdot(1-\beta) + 1$ and $S_{reduce} = S - n + 1 > S - S\cdot\beta = S\cdot(1-\beta)$. Therefore, $S_{reduce} = \lceil S\cdot(1-\beta)\rceil$. If $n/S = \beta$, then $S_{reduce} = S - n = S - S\cdot\beta = S\cdot(1-\beta)$. ■

Lemma 1–4 show how CVaR-TTR can be solved by CVaR-TTR-SR with a smaller searching space.

3) Order Scenario Reduction: Another scenario reduction strategy (order scenario reduction) is proposed by eliminating the second-stage decision variable, traversing order $\mathbf{q}$, under different scenarios to a first-stage decision variable. The reformulated model is a CVaR-based TTR model with scenario-order-free (CVaR-TTR-SOF). As a result, the traversing order is not related to scenarios. As the searching space is decreased, the CVaR-TTR-SOF provides an upper bound for CVaR-TTR.

Suppose both the scenario reduction strategy and order scenario reduction are considered. In that case, a reformulated CVaR-based TTR model with scenario reduction and scenario-order-free (CVaR-TTR-SR-SOF) is proposed. It provides the same upper bound for CVaR-TTR with a smaller searching space.

III. Computational Experiments

The reformulated MILP models for CVaR-TTR are solved by the commercial solver GUROBI 9.0.3, implemented in MATLAB R2018b using YALMIP as the modeling language with default parameter settings [11]. All experiments were carried out on a PC with an Intel Core i5-8265U CPU 1.60GHz and 8 GB internal memory.

A. Test Instances and Parameter Settings

The Beijing-–Tianjin intercity railway line from Beijing South to Tianjin is considered. It is a double-track railway. There are altogether 6 stations and 5 sections. 23 trains downstream from 6:00 to 9:00 are considered for the railway timetable, shown in Fig. 1. Some trains are heading to another railway corridor at Nancang. To distinguish trains in the timetable, the line width of trains is set differently.

The minimum running time of each section is shown in Table II. The additional times caused by starting and stopping are set to 2 min and 3 min, respectively. The dwell time for trains at stations is set based on the original timetable. It is set to 2 min for train stops at stations and no dwell time for pass-through stations, the origin stations, and destination stations. Other parameters are shown in Table III.

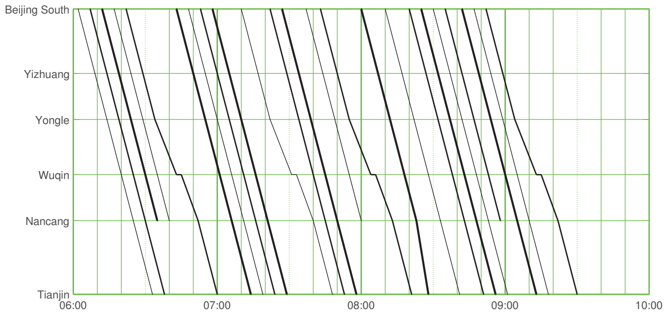

Fig. 1. Original timetable for Beijing–Tianjin intercity railway with 23 downstream trains within 3-h time horizon.

TABLE II
THE MINIMUM RUNNING TIME AT SECTIONS.

No.	Section	Time/min
1	Beijing South - Yizhuang	5
2	Yizhuang - Yongle	5
3	Yongle - Wuqin	6
4	Wuqin - Nancang	5
5	Nancang - Tianjin	5

TABLE III
PARAMETERS SETTINGS.

Parameters	Values
$\lvert I\rvert$	23
$\lvert J\rvert$	6
$\lvert K\rvert$	5
c_{ij}^s	0.5
e_{ij}^s	1
e_{ij}^e	1
α_i	1
β_i	5 (Train No. 3, 4, 13, 20); 6 otherwise
H_k	4 min
$\lvert S\rvert$	5

Two test instances are generated based on the time, place, and duration of the disruption as follows:

Instance No. 1: There is a section blockage at Yizhuang – Yongle, beginning from 6: 40. There are five scenarios for the duration of the disruption, which are 28 min, 29 min, 30 min, 31 min, and 33 min, with the same probability $p_s = 0.2$.

Instance No. 2: There is a section blockage at Yongle – Wuqin, beginning from 7: 30. There are five scenarios for the duration of the disruption, which are 24 min, 25 min, 26 min, 31 min, and 33 min, with the same probability $p_s = 0.2$.

B. Sensitivity Analysis for M

Since M is used in big-M constraints, i.e., headway constraints and train stop constraints, it should be greater than the headway between two adjacent trains when entering/leaving the station or the dwell time for one train. Therefore, M should be greater than 3 hours (180 min) with the duration of the disruption. To analyze the sensitivity of M, we test the CVaR-TTR-SR-SOF model on instance No. 1 with $\beta = 0.6$ with one run. The analysis result is shown in Table IV. From the table, we can conclude that $M = 500$ is suitable.

TABLE IV
PARAMETERS SETTINGS FOR M.

M/min	250	500	1000	2000	5000	10000	100000
Time/s	22.88	**14.1**	16.42	21.63	38.24	22.56	24.08

TABLE V
RESULTS FOR DIFFERENT MODELS (OBJECTIVE VALUE/RUNNING TIME (S)).

No.	β	CVaR-TTR	-SOF	-SR-SOF
1	0	933.60†	933.60/58.62	–
	0.2	970.25†	970.25/89.96	**970.25/49.51**
	0.4	1011.33†	1011.33/103.06	**1011.33/46.34**
	0.6	1060.00†	1041.50/95.51	**1041.50/15.27**
	0.8	1092.00/126.23	1092.00/101.48	**1092.00/12.95**
2	0	644.80/193.85	644.80/11.50	–
	0.2	681.75/75.34	681.75/10.99	**681.75/9.75**
	0.4	731.68/118.54	731.68/16.44	**731.68/12.18**
	0.6	**806.50/102.09**	810.00/28.97	810.00/8.04
	0.8	855.00/36.43	855.00/37.76	**855.00/7.29**

† GUROBI stopped after running for 10 min.
– The scenario reduction strategy is not applied for $\beta = 0$.

C. Result Analysis

The time limit for GUROBI is set to 10 min for TTR. Table V shows the objective value and running time for CVaR-TTR, CVaR-TTR-SOF (-SOF), and CVaR-TTR-SR-SOF (-SR-SOF) under two test instances. According to the table, the upper bound models (-SOF and -SR-SOF) can provide results efficiently with less time than CVaR-TTR. Meanwhile, the objective values for -SR-SOF are the same as those of -SR, and the running time decreases with the increase of the confidence level β. It is because the number of effective scenarios decreases with the increase of β. By reducing the number of scenarios, the number of variables and constraints in the model have been significantly reduced.

For instance No. 1, the CVaR-TTR model cannot be solved within 10 min when $0 \leq \beta \leq 0.6$. The corresponding -SR and SR-SOF models can be solved within 1–2 min for all instances. For instance No. 2, there is a difference between the upper bound model and the original model when $\beta = 0.6$. It shows that the traversing orders at undisrupted sections vary with different scenarios. The rescheduled timetables for instance No. 2 with $\beta = 0.6$ are shown in Figs. 2–5 with red lines for adjusted arrival and departure times. The objective value of the -SR-SOF model is 0.43% worse than that of CVaR-TTR, whereas the running time is 12.7 times better, which shows the effectiveness of -SR-SOF.

IV. CONCLUSION

This paper analyses the TTR problem in HSR with an uncertain duration of the section disruption. A two-stage stochastic programming model is proposed to minimize the CVaR of the total arrival and departure cost as a MINLP model. The model is linearized to a MILP model and effectively transformed into several models, including reducing the number of scenarios and traversing order scenarios. Computational experiments

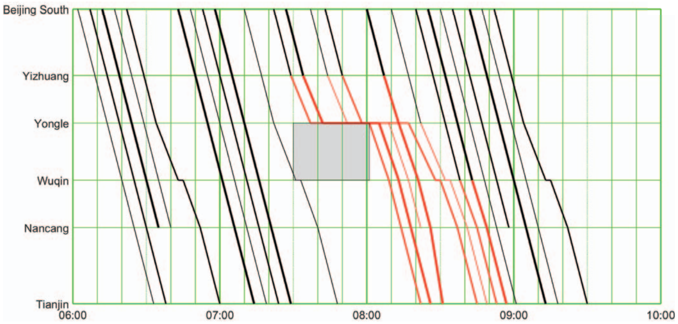

Fig. 2. Rescheduled timetable by CVaR-TTR with a duration of disruption equals 31 min when $\beta = 0.6$ for instance No. 2.

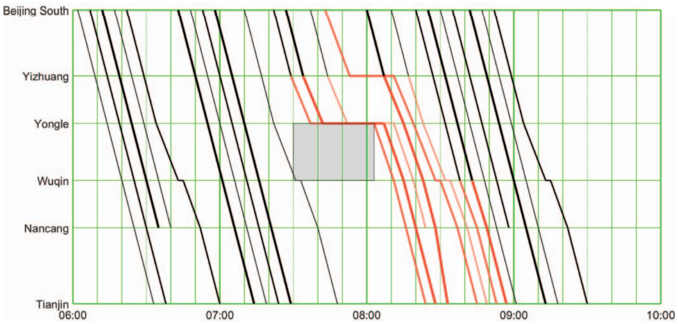

Fig. 3. Rescheduled timetable by CVaR-TTR with a duration of disruption equals 33 min when $\beta = 0.6$ for instance No. 2.

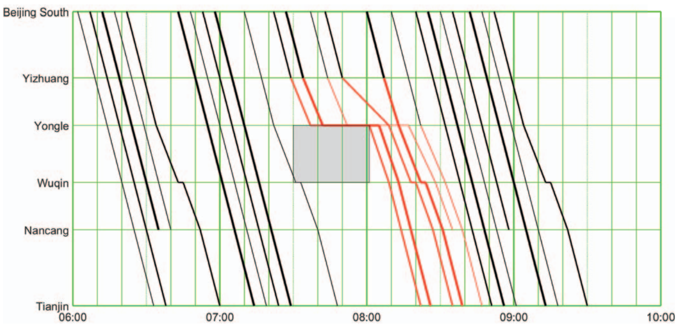

Fig. 4. Rescheduled timetable by CVaR-TTR-SR-SOF with a duration of disruption equals 31 min when $\beta = 0.6$ for instance No. 2.

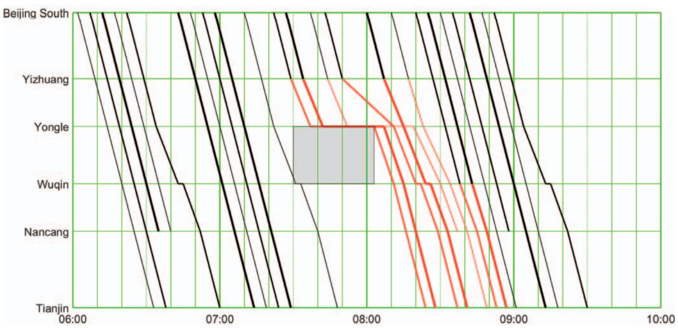

Fig. 5. Rescheduled timetable by CVaR-TTR-SR-SOF with a duration of disruption equals 33 min when $\beta = 0.6$ for instance No. 2.

demonstrate that the CVaR-TTR problem can be efficiently solved with optimal solutions and a few upper bound solutions.

In the future, new exact, heuristic, metaheuristic, reinforcement learning algorithms, and new reformulation can be applied to solve CVaR-TTR [12], [13]. The uncertain model can also be analyzed in other ways, e.g., preference information of DMs [14]. Besides, the multi-objective CVaR-TTR problem with more optimization objectives also needs further investigation [15].

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