High-Speed Railway Train Timetable Rescheduling in Case of a Stochastic Section Blockage (Paper ID: 4312)

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Introduction

- Train timetable rescheduling (TTR) is conducted when predefined train operations are affected by inevitable emergencies, e.g., train failure, natural disasters, etc.
- There are many uncertain features in actual train operations, which account for an efficient TTR method for increasing the train operation efficiency.
- Most of the studies consider the parameters for the emergencies are deterministic.
- This paper present a conditional value-at-risk (CVaR) based TTR model under uncertainty with stochastic disruption scenarios. Model transformation and approximation methods are applied to speed up the computation and provide an efficient upper bound.

Assumptions

- Disruption considered is a complete section blockage between two adjacent stations.
- There is only one disruption, whose duration is a stochastic variable with known distribution.
- Trains are allowed to arrive early at the station, but they should not depart early.

Objective Function

The total arrival and departure cost for all trains under scenario s is defined as:

$$D(\mathbf{x},\mathbf{q},\mathbf{y},s) = \sum_{i\in I}\sum_{j=\alpha_i}^{\beta_i} c_{ij}^s[o_{ij}^s(s) - x_{ij}^s(s)]^+ + e_{ij}^s[x_{ij}^s(s) - o_{ij}^s(s)]^+ + e_{ij}^e(x_{ij}^e(s) - o_{ij}^e) \quad (1)$$

where I denotes the set of trains. $\mathbf{x} = [x_{ij}^s(s), x_{ij}^e(s)]$, $\mathbf{q} = [q_{ilk}(s)]$ and $\mathbf{y} = [y_{ij}(s)]$ are the rescheduled time, order, and stop indicator. For S scenarios, the corresponding CVaR value is calculated by [1]:

$$\mathrm{CVaR}_\beta(D(\mathbf{x},\mathbf{q},\mathbf{y},s)) = \min_{\alpha\in\mathbb{R}}\left\{\alpha + \frac{1}{1-\beta}\sum_{s\in S} p_s\,[D(\mathbf{x},\mathbf{q},\mathbf{y},s) - \alpha]^+\right\} \quad (2)$$

where p_s is the probability of scenario s, $\alpha\in\mathbb{R}$. S denotes the set of scenarios.

Constraints

- Dwell Time Constraints
- Running Time Constraints
- Headway Constraints
- Close-to-Favorite-Schedule Constraints
- Initial Rescheduling Time Constraints
- Arrival Time Constraints
- Traversing Order Constraints: For two adjacent trains, either one can traverse at a section before the other.

$$q_{ilk}(s) + q_{ilk}(s) = 1 \quad (3)$$

$$q_{ilk^*}(s=1) = \cdots = q_{ilk^*}(s=S) \quad (4)$$

  where $q_{ilk^*}(s)$ is a first-stage decision variable remains the same under different scenarios. In the second stage, the rescheduled arrival, departure time, traversing order in other sections, and train stop indicator are decided when the random quantities can be observed.
- Train Stop Constraints

CVaR-based TTR model and Its Reformulations

- The CVaR-based TTR model is formulated to minimize the CVaR of the rescheduling cost (2) under several constraints, that is: min $\mathrm{CVaR}_\beta(D(\mathbf{x},\mathbf{q},\mathbf{y},s))$. Since there are continuous real variables $\mathbf{x}$, 0-1 variables $(\mathbf{q},\mathbf{y})$, and nonlinear terms (minimax function) in (2), the proposed CVaR-based TTR model belongs to mixed-integer nonlinear programming (MINLP).
- Model Reformulation
  - **Linearization.** The nonlinear terms (minimax function) are tackled by introducing auxiliary variables.
  - **Scenario Reduction.** When β is greater than 0, the corresponding scenarios related with β are used for calculation, rather than all the scenarios. Therefore, a scenario reduction strategy is proposed to speed up the model according to this problem-specific knowledge.
  - **Order Scenario Reduction.** Another scenario reduction strategy (order scenario reduction) is proposed by eliminating the second-stage decision variable, traversing order $\mathbf{q}$, under different scenarios to a first-stage decision variable. The reformulated model is a CVaR-based TTR model with scenario-order-free (CVaR-TTR-SOF). As a result, the traversing order is not related to scenarios. As the searching space is decreased, the CVaR-TTR-SOF provides an upper bound for CVaR-TTR.
  - Suppose both the **scenario reduction strategy** and **order scenario reduction** are considered. In that case, a reformulated CVaR-based TTR model with scenario reduction and scenario-order-free (CVaR-TTR-SR-SOF) is proposed. **It provides the same upper bound for CVaR-TTR with a smaller searching space.**

Computational Experiments

- The Beijing-–Tianjin intercity railway line from Beijing South to Tianjin is considered. It is a double-track railway. There are altogether 6 stations and 5 sections. 23 trains downstream from 6:00 to 9:00 are considered for the railway timetable. Some trains are heading to another railway corridor at Nancang. To distinguish trains in the timetable, the line width of trains is set differently. Two test instances are generated based on the time, place, and duration of the disruption under five scenarios.
- The time limit for GUROBI is set to 10 min for TTR. Table 1 shows the objective value and running time for CVaR-TTR, CVaR-TTR-SOF (-SOF), and CVaR-TTR-SR-SOF (-SR-SOF) under two test instances. According to the table, the upper bound models (-SOF and -SR-SOF) can provide results efficiently with less time than CVaR-TTR. Meanwhile, the objective values for -SR-SOF are the same as those of -SR, and the running time decreases with the increase of the confidence level β. It is because the number of effective scenarios decreases with the increase of β. **By reducing the number of scenarios, the number of variables and constraints in the model have been significantly reduced.**
- For instance No. 1, the CVaR-TTR model cannot be solved within 10 min when $0 \le \beta \le 0.6$. The corresponding -SR and SR-SOF models can be solved within 1–2 min for all instances. For instance No. 2, there is a difference between the upper bound model and the original model when $\beta = 0.6$. It shows that the traversing orders at undisrupted sections vary with different scenarios. The rescheduled timetables for instance No. 2 with $\beta = 0.6$ and disruption duration equals 33 min are shown in Figs. 1–2 with red lines for adjusted arrival and departure times. **The objective value of the -SR-SOF model is 0.43% worse than that of CVaR-TTR, whereas the running time is 12.7 times better, which shows the effectiveness of -SR-SOF.**

Figures and Tables

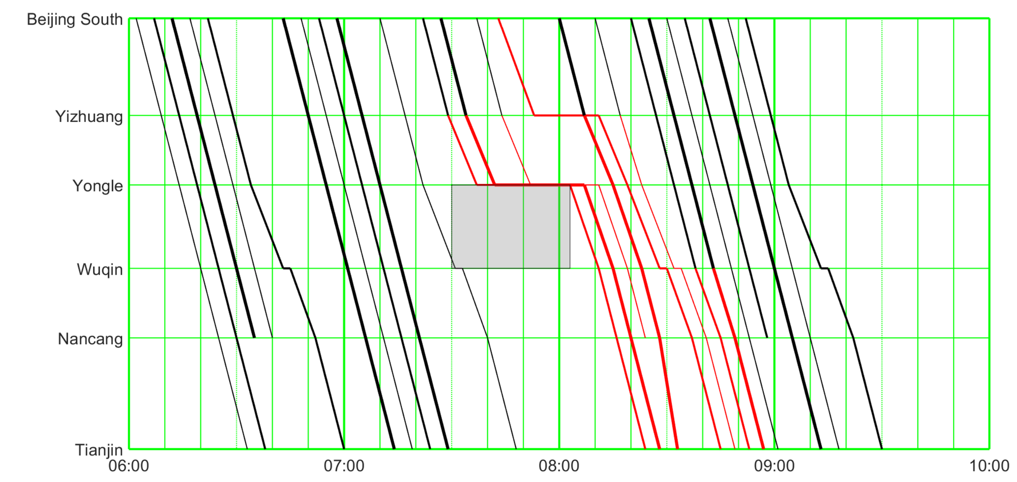

Fig. 1: Rescheduled timetable by CVaR-TTR with a duration of disruption equals 33 min when $\beta = 0.6$ for instance No. 2.

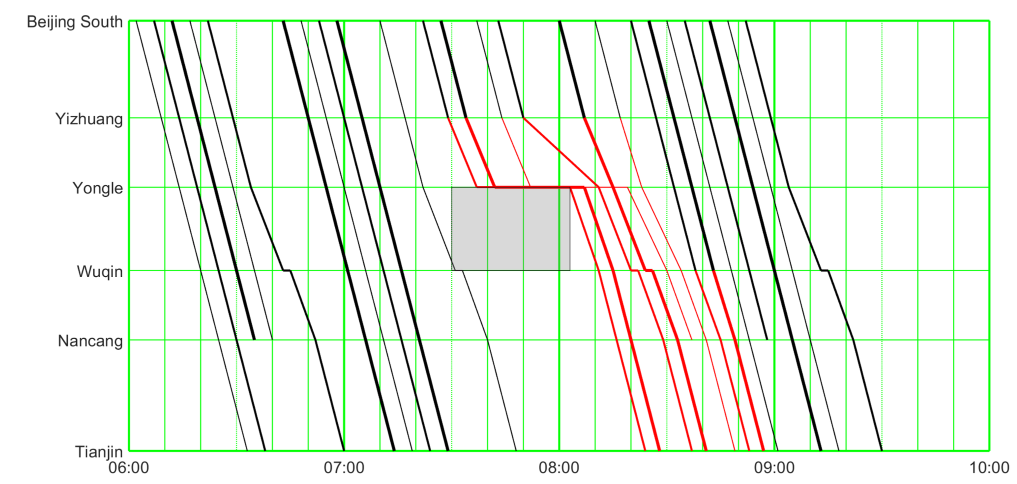

Fig. 2: Rescheduled timetable by CVaR-TTR-SR-SOF with a duration of disruption equals 33 min when $\beta = 0.6$ for instance No. 2.

No.	β	CVaR-TTR	-SOF	-SR-SOF
1	0	933.60†	933.60/58.62	–
	0.2	970.25†	970.25/89.96	**970.25/49.51**
	0.4	1011.33†	1011.33/103.06	**1011.33/46.34**
	0.6	1060.00†	1041.50/95.51	**1041.50/15.27**
	0.8	1092.00/126.23	1092.00/101.48	**1092.00/12.95**
2	0	644.80/193.85	644.80/11.50	–
	0.2	681.75/75.34	681.75/10.99	**681.75/9.75**
	0.4	731.68/118.54	731.68/16.44	**731.68/12.18**
	0.6	**806.50/102.09**	810.00/28.97	810.00/8.04
	0.8	855.00/36.43	855.00/37.76	**855.00/7.29**

† GUROBI stopped after running for 10 min.
– The scenario reduction strategy is not applied for $\beta = 0$.

Table 1: Results for different models (objective value/running time (s)).

Conclusion

A two-stage stochastic programming model is proposed to minimize the CVaR of the total arrival and departure cost. The model is linearized to a MILP model and effectively transformed into several models. The problem can be efficiently solved with optimal solutions and a few upper bound solutions.

References

[1] R. T. Rockafellar, S. Uryasev et al., *Optimization of conditional valueat-risk*, Journal of Risk, vol. 2, pp. 21–42, 2000.