A Comparative Study on Evolutionary Algorithms for High-Speed Railway Train Timetable Rescheduling Problem

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Outline

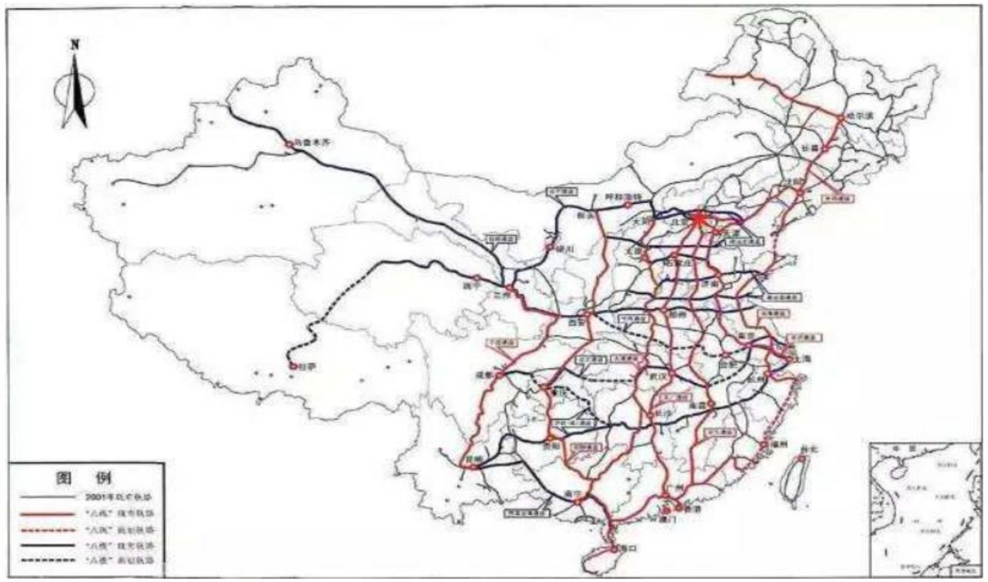
- Introduction
- Model Formulation
- Evolutionary Algorithms for Train Timetable Rescheduling
- Computational Experiments
- Concluding Remarks

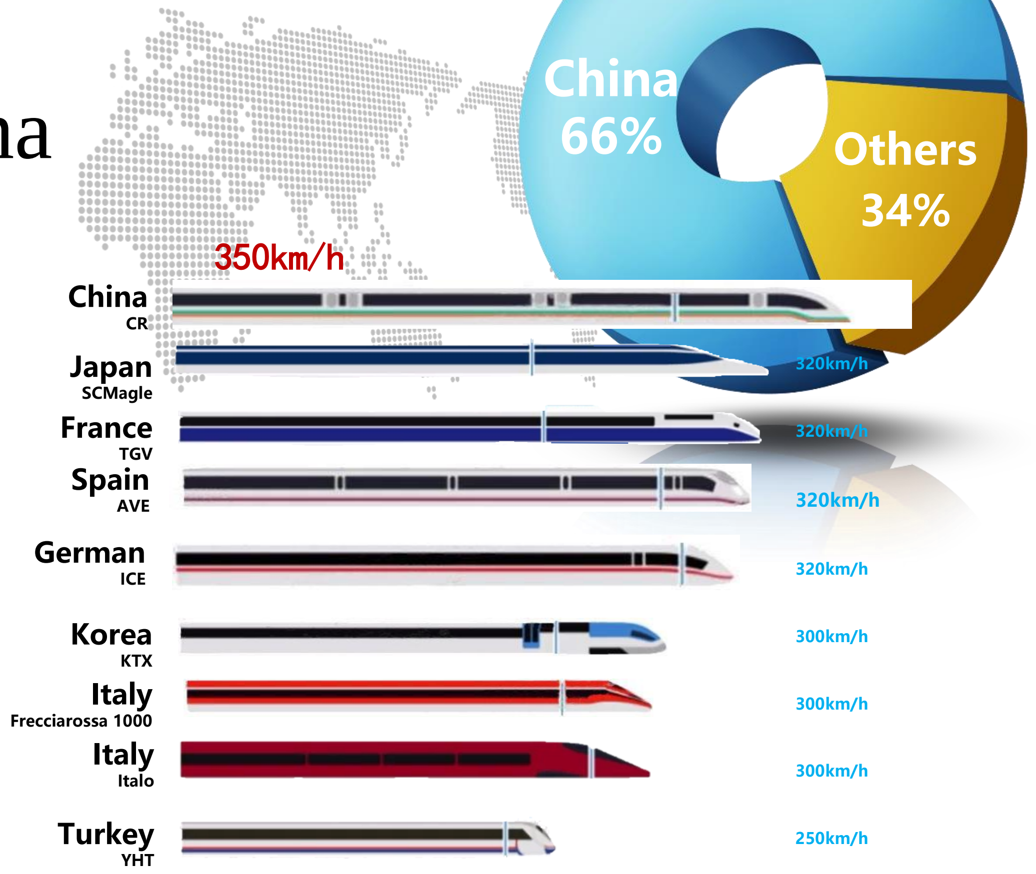
Introduction

China High-Speed Railway (HSR)——37900 kilometers

Operation as a network only in China

China High-Speed Railway Network





It is a great challenge to keep the HSR operate punctually

- Large network size
- High operation speed
- High traffic density
- Large amount of operation
- Complex transportation organization
- Diversified travel demand

Train Timetable Rescheduling is the key issue for emergency decision under disruption

- If the dispatching is not reasonable, once an emergency occurs, it is easy to cause a large area of train delay and other serious consequences, bringing inconvenience to passengers and reducing the operation efficiency of high-speed railway

2021.05
Beijing-Tianjin intercity high-speed railway with severe delay since overhead line with foreign matter

2018.12
Heavy snow cause multiple train delay in Changsha South Station

How to propose a simple and effective rescheduling model and a fast solution algorithm has become an urgent need for the efficient operation of high-speed railway in China!

Train dispatching system is the "brain" and "commander" of high-speed railway system

Real Application

Mainly handled by dispatchers based on their experience under emergencies

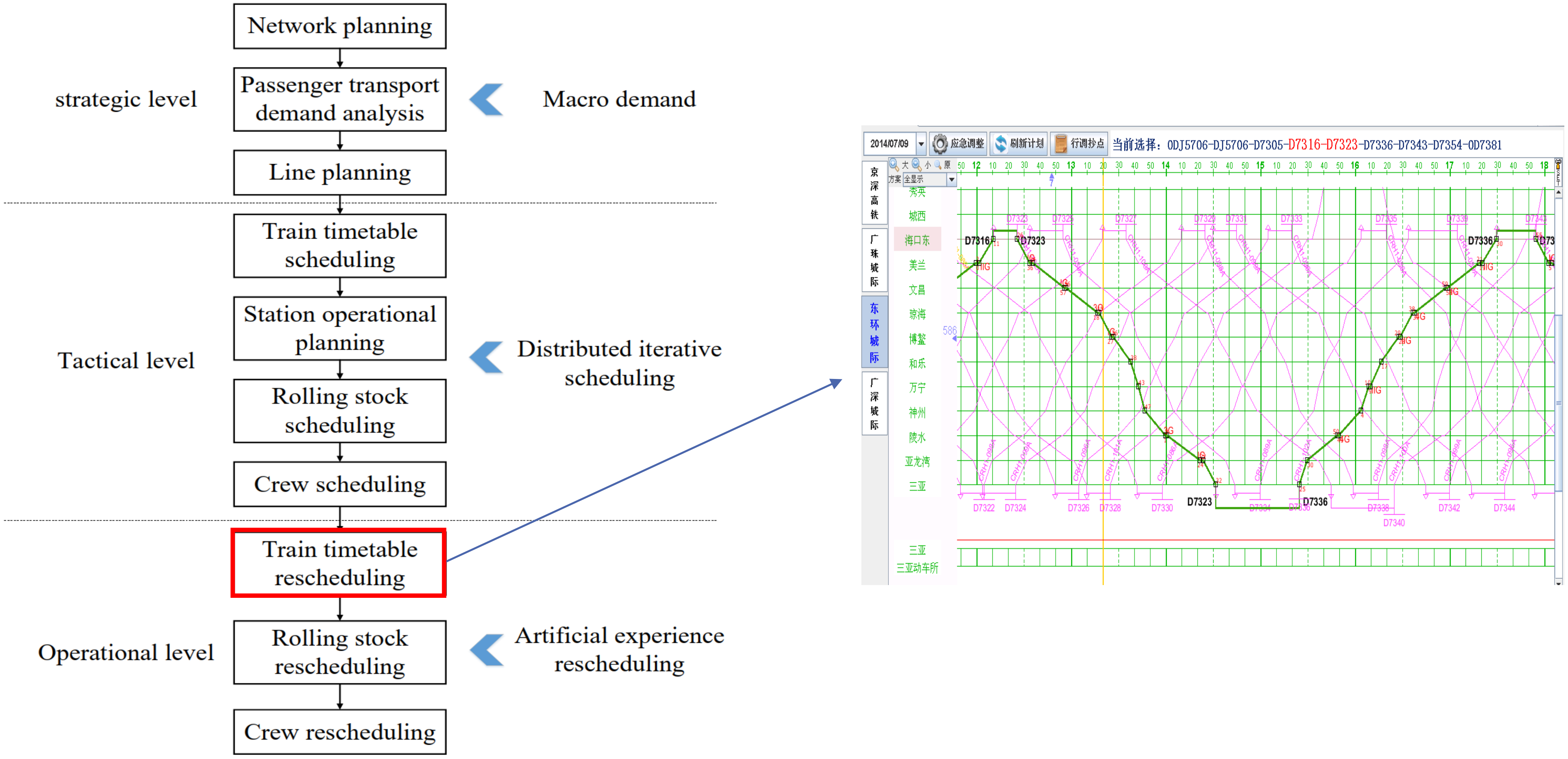
Manual scheduling decision is not optimal decision, which cannot guarantee high efficiency and precise operation

Theoretical research

①Formulate mixed integer linear programming models
②Use exact method, metaheuristics, or AI technique

① NP-hard
② Time consuming and suboptimal

Different levels in train scheduling



Paper Contribution

- The high-speed railway train timetable rescheduling problem with a complete station blockage is proposed and modeled as a MILP problem.
- An effective permutation encoding method is proposed for the TTR problem, and a rule-based decoding method is designed to obtain a new schedule. These encoding and decoding methods can manage the entire constraints and guarantee the feasibility of the solution.
- Several evolutionary algorithms are used for solving TTR. Experimental results show that SaDE can efficiently solve most of the test instances compared with other algorithms.

Model Formulation

Decision Variables

Symbol	Description
$t_{i,s}^{a}$	the actual arrival time of train i at station s
$t_{i,s}^{d}$	the actual departure time of train i at station s
$q_{i,j,(s,s+1)}$	the actual traversing order, 1 if train i traverses on section $(s,\ s+1)$ before train j; 0 otherwise
$y_{i,s}$	the actual train stop indicator, 1 if train i stops at station $(s,\ s+1)$; 0 otherwise

$$t_{i,s}^{a}, t_{i,s}^{d} \geq 0 \qquad q_{i,j,(s,s+1)}, y_{i,s} \in \{0,1\}$$

Formulation

Objective function

- Minimize the total delay time, including the delay arrival and departure time of each train at all the stations

$$\min \sum_{i\in T}\sum_{s\in S} w_i(t_{i,s}^a - T_{i,s}^a + t_{i,s}^d - T_{i,s}^d)$$

Formulation

Constraints

- Minimum dwelling time constraints
- Minimum running time constraints
- Headway constraints for departure headway and arrival headway
- Traverse order constraint of two trains in a section
- The arrival and departure times for the unaffected trains are equal to the original timetable
- No trains are allowed to arrive at stations during the disruption
- Timetable constraints that restrict trains are not allowed to arrive and depart from stations before the original arrival and departure time
- The actual traversing orders of all trains are equal to the traversing orders in their first section
- Train stop indicator constraints

Formulation

Constraints

- Minimum dwelling time constraints
- Minimum running time constraints
- Headway constraints for departure headway and
- Traverse order constraint of two trains in a sectio
- The arrival and departure times for the unaffecte timetable
- No trains are allowed to arrive at stations during
- Timetable constraints that restrict trains are not a stations before the original arrival and departure
- The actual traversing orders of all trains are equa first section
- Train stop indicator constraints

$$\text{s.t. } t^d_{i,s} - t^a_{i,s} \ge d_{i,s}\ \forall i \in T; s \in S \quad (2)$$

$$t^a_{i,s+1} - t^d_{i,s} \ge r^{min}_{i,(s,s+1)} + r^s_{i,(s,s+1)} y_{i,s} + r^e_{i,(s,s+1)} y_{i,s+1}\ \forall i \in T; s \in S \backslash D(i) \quad (3)$$

$$t^d_{j,s} - t^d_{i,s} \ge h_{(s,s+1)} q_{i,j,(s,s+1)} - M(1 - q_{i,j,(s,s+1)})\ \forall i,j \in T; i \neq j; s \in S \backslash D(i) \quad (4)$$

$$t^a_{j,s+1} - t^a_{i,s+1} \ge h_{(s,s+1)} q_{i,j,(s,s+1)} - M(1 - q_{i,j,(s,s+1)})\ \forall i,j \in T; i \neq j; s \in S \backslash D(i) \quad (5)$$

$$q_{i,j,(s,s+1)} + q_{j,i,(s,s+1)} = 1\ \forall i,j \in I; i \neq j; s \in S \backslash D(i) \quad (6)$$

$$t^a_{i,s} = T^a_{i,s}\ \forall i \in T; s \in S : T^a_{i,s} \le H^s_{dis} \quad (7)$$

$$t^d_{i,s} = T^d_{i,s}\ \forall i \in T; s \in S : T^d_{i,s} \le H^s_{dis} \quad (8)$$

$$t^a_{i,s^*} \ge H^s_{dis} + D_{dis}\ \forall i \in T : H^s_{dis} \le T^a_{i,s^*} \le H^s_{dis} + D_{dis} \quad (9)$$

$$t^a_{i,O(i)} = t^d_{i,O(i)}\ \forall i \in T \quad (10)$$

$$t^a_{i,s} \ge T^a_{i,s}\ \forall i \in T; s \in S \quad (11)$$

$$t^d_{i,s} \ge T^d_{i,s}\ \forall i \in T; s \in S \quad (12)$$

$$q_{i,j,(O(i),O(i)+1)} = q_{i,j,(s,s+1)}\ \forall i,j \in T; i \neq j; s \in S \backslash \{O(i), D(i)\} \quad (13)$$

$$y_{i,s} \le t^d_{i,s} - t^a_{i,s}\ \forall i \in T; s \in S \backslash \{O(i), D(i)\} \quad (14)$$

$$y_{i,s} \ge \frac{t^d_{i,s} - t^a_{i,s}}{M}\ \forall i \in T; s \in S \backslash \{O(i), D(i)\} \quad (15)$$

$$y_{i,s} \ge Y_{i,s}\ \forall i \in T; s \in S \backslash \{O(i), D(i)\} \quad (16)$$

$$y_{i,s} = Y_{i,s}\ \forall i \in T; s \in \{O(i), D(i)\} \quad (17)$$

Formulation

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$$t^a_{i,s+1} - t^d_{i,s} \geq r^{min}_{i,(s,s+1)} + r^s_{i,(s,s+1)} y_{i,s} + r^e_{i,(s,s+1)} y_{i,s+1}$$
$$\forall i \in T; s \in S \backslash D(i) \quad (3)$$

$$t^d_{j,s} - t^d_{i,s} \geq h_{(s,s+1)} q_{i,j,(s,s+1)} - M(1 - q_{i,j,(s,s+1)})$$
$$\forall i,j \in T; i \neq j; s \in S \backslash D(i) \quad (4)$$

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$$t^a_{i,s} = T^a_{i,s} \ \forall i \in T; s \in S : T^a_{i,s} \leq H^s_{dis} \quad (7)$$

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$$t^a_{i,s*} \geq H^s_{dis} + D_{dis} \ \forall i \in T : H^s_{dis} \leq T^a_{i,s*} \leq H^s_{dis} + D_{dis} \quad (9)$$

$$t^a_{i,O(i)} = t^d_{i,O(i)} \ \forall i \in T \quad (10)$$

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$$q_{i,j,(O(i),O(i)+1)} = q_{i,j,(s,s+1)}$$
$$\forall i,j \in T; i \neq j; s \in S \backslash \{O(i), D(i)\} \quad (13)$$

$$y_{i,s} \leq t^d_{i,s} - t^a_{i,s} \ \forall i \in T; s \in S \backslash \{O(i), D(i)\} \quad (14)$$

$$t^d_{i,s} - t^a_{i,s}$$

$$y_{i,s} = Y_{i,s} \ \forall i \in T; s \in \{O(i), D(i)\} \quad (17)$$

The problem is an **mixed integer linear programming** problem which belongs to **NP-hard**

Evolutionary Algorithms for Train Timetable Rescheduling

Encoding and Decoding

- Using permutation-based encoding instead of real-coded encoding
- Real-coded encoding

T: Set of trains S: Set of stations

$$[t^a_{1,1}, t^d_{1,1}, t^a_{1,2}, t^d_{1,2}, \ldots, t^a_{i,s}, t^d_{i,s}, \ldots, t^a_{|T|,|S|}, t^d_{|T|,|S|}], i \in T, s \in S, 1 \le t^a_{i,s}, t^d_{i,s} \le 1440$$

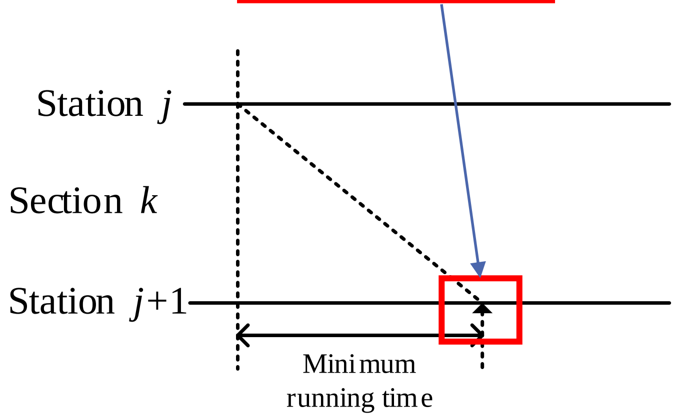
  - Dimension: $2|T||S|$ Solution space: $1440^{2|T||S|}$ (for integer arrival/departure time)
- Permutation-based encoding

$$[p_1, p_2, \ldots, p_i, \ldots, p_{|T|}], i \in T, p_i \in \{1, \ldots, |T|\}, p_i \ne p_j : i \ne j$$

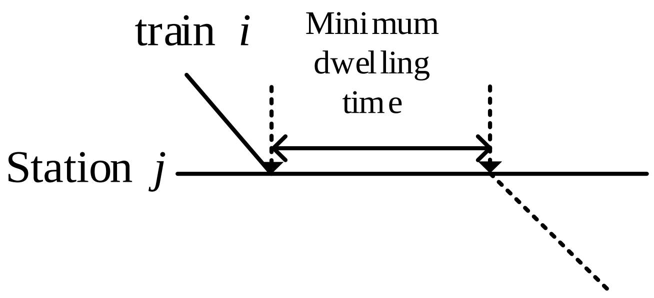
  - Dimension: $|T|$ Solution space: $|T|!$
- The dimension and solution space is much smaller in permutation-based encoding
- There are unfeasible region in real-coded encoding, constraints handling should be designed

Encoding and Decoding

- Obtain the actual arrival time and departure time through the decoding procedure
  - Traversing order is obtained through the permutation-based encoding
  - Decide arrival time and departure time satisfying different constraints



Minimum running time constraints



Minimum dwelling time constraints

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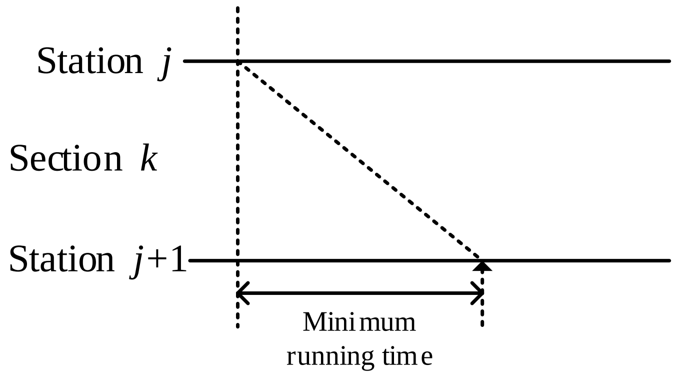

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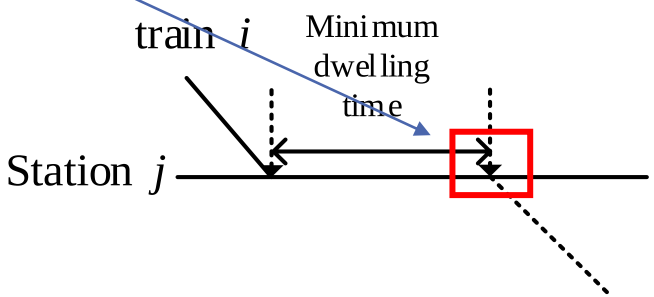

Minimum dwelling time constraints

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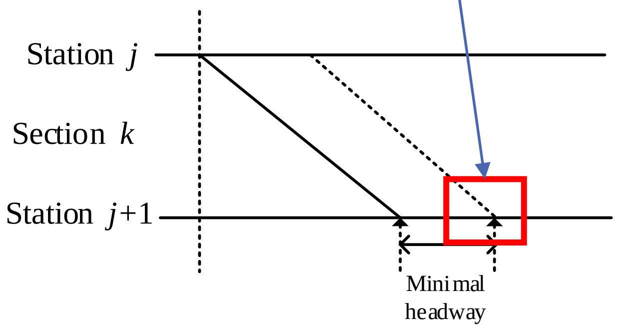
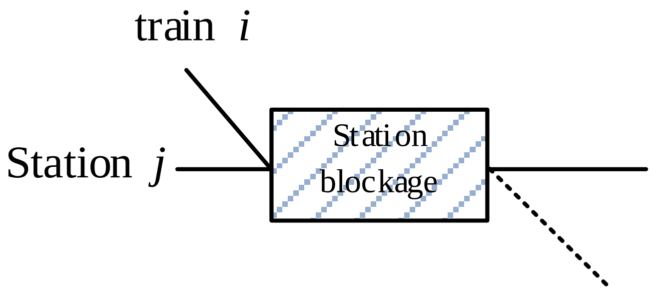


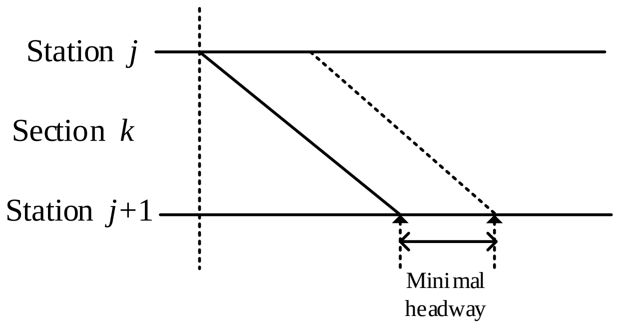
Headway constraints



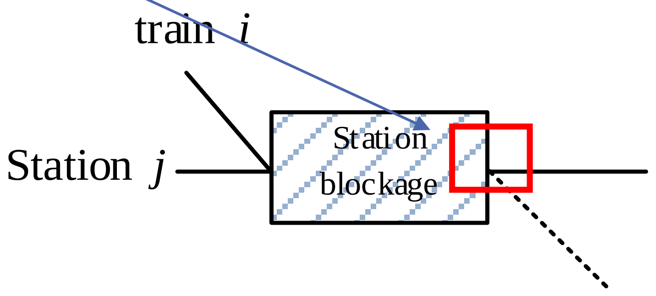
Depart after disruption ends

Encoding and Decoding

- Obtain the actual arrival time and departure time through the decoding procedure
  - Traversing order is obtained through the permutation-based encoding
  - Decide arrival time and departure time satisfying different constraints



Headway constraints



Depart after disruption ends

Encoding and Decoding

Algorithm 1 Decoding Procedure

Input: The original timetable information; The disruption information; The set of affected trains T_{dis}; Scheduling order of the trains $\mathbf{p} = [p_i]_{1\times|T|}$

Output: The actual arrival time $t^a_{i,s}$ and departure time $t^d_{i,s}$

for $i = 1$ to $|T| - |T_{dis}|$ **do**
    for $s = O(i)$ to $D(i)$ **do**
        $t^a_{i,s} = T^a_{i,s}$; $t^d_{i,s} = T^d_{i,s}$;
    end for
end for
for $i = |T| - |T_{dis}| + 1$ to $|T|$ **do**
    if $i = |T| - |T_{dis}| + 1$ **then**
        $t^a_{p_i,O(p_i)} = H^s_{dis} + D_{dis}$;
        $t^d_{p_i,O(p_i)} = t^a_{p_i,O(p_i)}$;
    else
        $t^a_{p_i,O(p_i)} = \max(t^a_{p_{i-1},O(p_{i-1})} + h_{(O(p_i),O(p_i)+1)}, T^a_{p_i,O(p_i)})$;
        $t^d_{p_i,O(p_i)} = \max(t^a_{p_i,O(p_i)} + d_{p_i,O(p_i)}, T^d_{p_i,O(p_i)})$;
    end if
    $y_{p_i,O(p_i)} = Y_{p_i,O(p_i)}$;
    for $s = O(i) + 1$ to $D(i)$ **do**
        $y_{p_i,s} = Y_{p_i,s}$;
        $t^a_{p_i,s} = \max(t^d_{p_i,s-1} + r^{min}_{p_i,(s-1,s)} + y_{p_i,s-1} r^s_{p_i,(s-1,s)} + y_{p_i,s} r^e_{p_i,(s-1,s)}, T^a_{p_i,s})$;
        $t^a_{p_i,s} = \max(t^a_{p_i,s}, t^a_{p_{i-1},s} + h_{(s-1,s)})$;
        $t^d_{p_i,s} = \max(t^a_{p_i,s} + d_{p_i,s}, T^d_{p_i,s})$;
        if $s < D(p_i)$ **then**
            $t^d_{p_i,s} = \max(t^d_{p_i,s}, t^d_{p_{i-1},s} + h_{(s,s+1)})$;
            if $\mathrm{sgn}(t^d_{p_i,s} - t^a_{p_i,s}) > y_{p_i,s}$ **then**
                $t^a_{p_i,s} = \min(t^d_{p_i,s-1} + r^{min}_{p_i,(s-1,s)} + y_{p_i,s-1} r^s_{p_i,(s-1,s)} + r^e_{p_i,(s-1,s)}, t^d_{p_i,s})$;
                $y_{p_i,s} = \mathrm{sgn}(t^d_{p_i,s} - t^a_{p_i,s})$;
            end if
        end if
    end for
end for
return

Evolutionary Algorithms

- A Dual-Model Estimation of Distribution Algorithm (DM-EDA)
- Self-adaptive Differential Evolution (SaDE)
- Comprehensive Learning Particle Swarm Optimizer (CLPSO)
- Covariance Matrix Adaptation Evolution Strategy (CMA-ES)

Evolutionary Algorithms

- A Dual-Model Estimation of Distribution Algorithm (DM-EDA)
  - It estimates the overall distribution of the parent solutions and updates a probabilistic model with the superior individuals
  - New solutions are sampled from the model
  - Node histogram model (NHM) and edge histogram model (EHM) are selected for permutation-based optimization problem
  - Truncation selection and restart strategy are used

DM-EDA is designed to search in discrete space

Evolutionary Algorithms

- Self-adaptive Differential Evolution (SaDE)
  - It uses a self-adaptive method to choose trial vector generation strategies and control parameter values
- Comprehensive Learning Particle Swarm Optimizer (CLPSO)
  - Each dimension of a particle learns from the best corresponding dimension of the particle
- Covariance Matrix Adaptation Evolution Strategy (CMA-ES)
  - It also uses a probability model to obtain new solutions
  - It samples solutions from a multivariate normal distribution

All above algorithms are designed to search in continuous space

Evolutionary Algorithms

- A Dual-Model Estimation of Distribution Algorithm (DM-EDA)
- Self-adaptive Differential Evolution (SaDE)
- Comprehensive Learning Particle Swarm Optimizer (CLPSO)
- Covariance Matrix Adaptation Evolution Strategy (CMA-ES)
- **Random Key Algorithm** for algorithms designed to search in continuous space (SaDE, CLPSO, and CMA-ES)

real vector (3.5, 2.4, 1.6, 0.5, 4.1) —ranking→ permutation (4, 3, 2, 1, 5)

Computational Experiments

Computational Experiments

- The Beijing–Tianjin intercity HSR timetable
- 6 stations and 5 sections
- 40 trains downstream from 6:00 to 12:00
- Dwell time: 2min
- Minimum running time of each section are 5, 5, 6, 5, 5 (min), respectively
- Additional times caused by starting and stopping are 2 min and 3 min
- Minimal headway: 4min

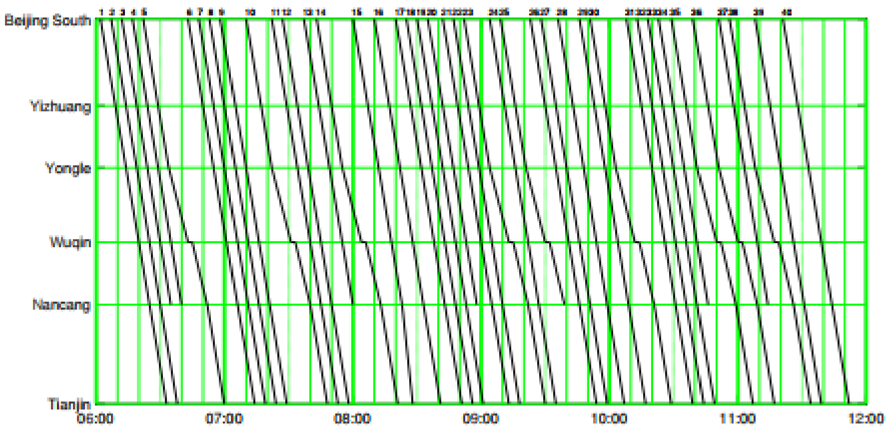


Fig. 1. Original timetable for Beijing–Tianjin intercity railway with 40 downstream trains within 6-h time horizon.

Computational Experiments

- 8 test instances from 2 cases on train weights
  - Case 1: The weight values of trains are set to 1.
  - Case 2: The weight values of trains are generated as uniformly distributed random integers in a range between 1 to 10.
- $|T|$ is the total trains considered
- D_{dis} is the disruption duration

Table 3. Setting of the two basic parameters for the test instances.

No.	$\|T\|$	D_{dis} (min)	No.	$\|T\|$	D_{dis} (min)
1, 5	15	30	2, 6	20	50
3, 7	30	70	4, 8	40	90

Computational Experiments

Table 4. Results of the comparison between DM-EDA, SaDE, CLPSO, and CMA-ES.

No.	DM-EDA	SaDE	CLPSO	CMA-ES	CPLEX
1	**1628.0000 ± 0.0000**‡	**1628.0000 ± 0.0000**‡	**1628.0000 ± 0.0000**‡	**1628.0000 ± 0.0000**‡	1628.0000‡
2	**3874.0000 ± 0.0000**‡	**3874.0000 ± 0.0000**‡	**3874.0000 ± 0.0000**‡	**3874.0000 ± 0.0000**‡	3874.0000‡
3	7570.8000 ± 34.5522	**7272.8000 ± 7.5226**	7274.4000 ± 7.6116	7284.3000 ± 0.7327	7268.0000†
4	12539.2000 ± 55.0154	**12070.0000 ± 0.0000**‡	12072.1000 ± 3.3388	12081.7000 ± 13.2709	12070.0000†
5	6462.0000 ± 0.0000	**6126.0000 ± 0.0000**‡	**6126.0000 ± 0.0000**‡	**6126.0000 ± 0.0000**‡	6126.0000‡
6	15386.0000 ± 0.0000	**14810.0000 ± 0.0000**‡	**14810.0000 ± 0.0000**‡	15060.6000 ± 695.6067	14810.0000‡
7	31475.0500 ± 684.5033	**26874.6000 ± 8.0026**	26875.3000 ± 8.3168	27177.0000 ± 330.6453	26872.0000‡
8	59492.1000 ± 1055.7585	**43125.0000 ± 10.7508**	43636.0000 ± 157.0169	43697.0000 ± 599.0109	43128.0000†

† CPLEX stopped after running for one hour.
‡ Optimal value.

- In five instances (No. 1, 2, 4, 5, and 6), the results of SaDE equal that of CPLEX.
- Moreover, for instances No. 3 and 7, the results of SaDE are only slightly larger (0.07% and 0.01%) than that of CPLEX (within one hour).
- In instance No. 8, the result of SaDE is better than that of CPLEX (within one hour).

Computational Experiments

- Converge curves of the four EAs in instances No. 3, 4, 7, and 8
- The curves are zoomed in some areas for better visualization
- CMA-ES converges faster than other algorithms
- SaDE converges second but provides better results.

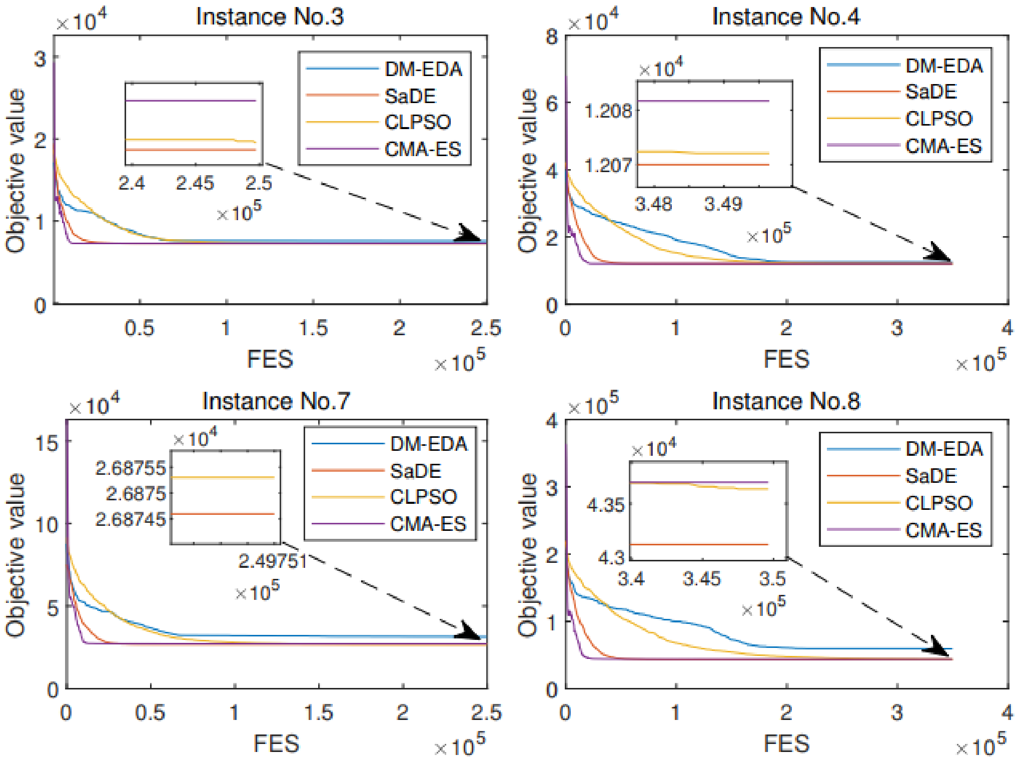


Fig. 2. Convergence curves of the proposed DM-EDA, SaDE, CLPSO, and CMA-ES for several test instances.

Computational Experiments

Table 5. Runtime performance of different algorithms (sec.).

No.	DM-EDA	SaDE	CLPSO	CMA-ES	CPLEX
1	5.7372 ± 0.4578	8.1800 ± 0.6489	3.7526 ± 0.2992	2.2386 ± 0.3614	10.3855
2	10.0875 ± 0.6872	11.9927 ± 0.6156	5.5700 ± 0.3859	3.0539 ± 0.2743	64.7492
3	24.8920 ± 0.9677	19.9135 ± 1.2381	11.2691 ± 1.8045	6.0701 ± 1.0042	–
4	47.5454 ± 1.8224	30.0148 ± 2.1255	17.3618 ± 0.6499	9.6739 ± 0.1859	–
5	5.1246 ± 0.5452	8.1143 ± 1.1712	3.7058 ± 0.6896	1.8761 ± 0.3053	10.5488
6	10.2779 ± 1.2930	12.3090 ± 2.0349	6.0593 ± 0.7121	2.7641 ± 0.1174	30.5911
7	24.8921 ± 0.8240	20.0972 ± 1.1450	11.3079 ± 1.3316	6.2739 ± 1.1048	2861.8612
8	49.8737 ± 3.1044	31.1891 ± 2.9282	17.4095 ± 0.8151	10.4551 ± 1.6275	–

– CPLEX cannot find optimal value after running for one hour.

- The result shows that DM-EDA takes the longest time compared with the other EAs.
- All instances can be solved within one minute.
- The running time for CPLEX increases. For some instances, it is more than 1 hour.

Concluding Remarks

- The high-speed railway TTR problem is formulated as a MILP problem.
- Four EAs are designed to solve TTR.
- A novel encoding and decoding method are specially designed.
- Obtained optimal/suboptimal solutions within one minute.

Future Research

- Consider situations with more types of trains (e.g., trains with different prefixes including G, C, D).
- Consider reordering in other stations.
- Consider the uncertainties in the dynamic environment.

Thank you for your attention!

Q&A

Motivation

- High-speed railway (HSR) may face inevitable emergencies, e.g., infrastructure failure, train failure, natural disasters.
- When the scale of the problem is getting larger, using the CPLEX solver will cost much time, which may exceed the time limit.
- Unlike past works use real-encoding based metaheuristics.

We introduce a novel train timetable rescheduling problem with **a complete station blockage** as an **mixed integer linear programming** and considers **an effective permutation-based metaheuristics** to solve the problem with **near-optimal/optimal** solutions in **real-time**.