Evolutionary Multi-Objective Optimization for High-Speed Railway Train Timetable Rescheduling with Optimal/Suboptimal Solutions into Initial Population

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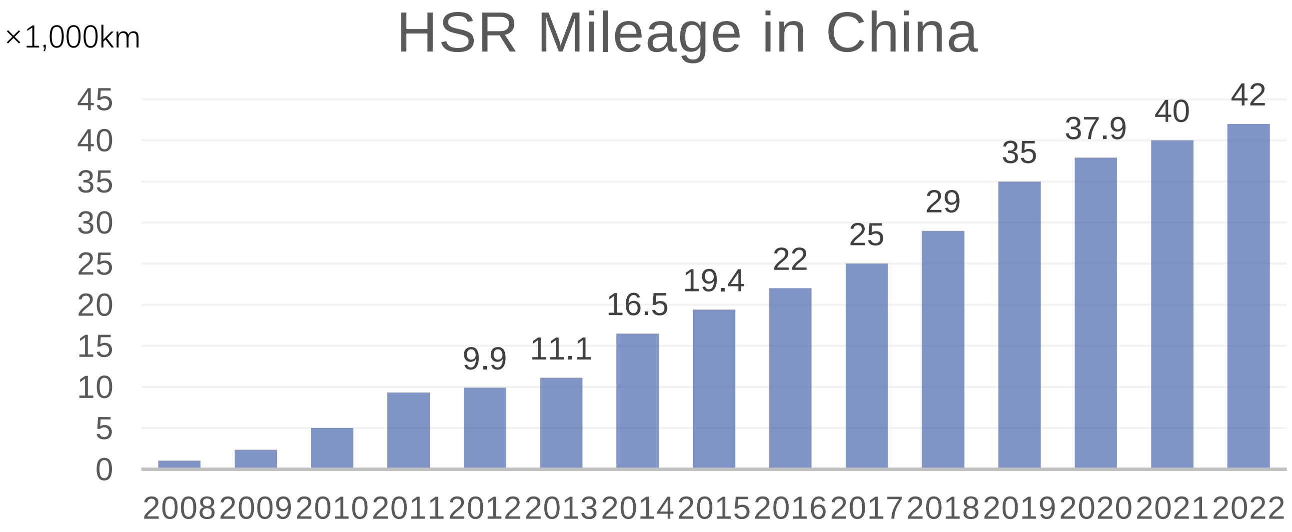
Outline

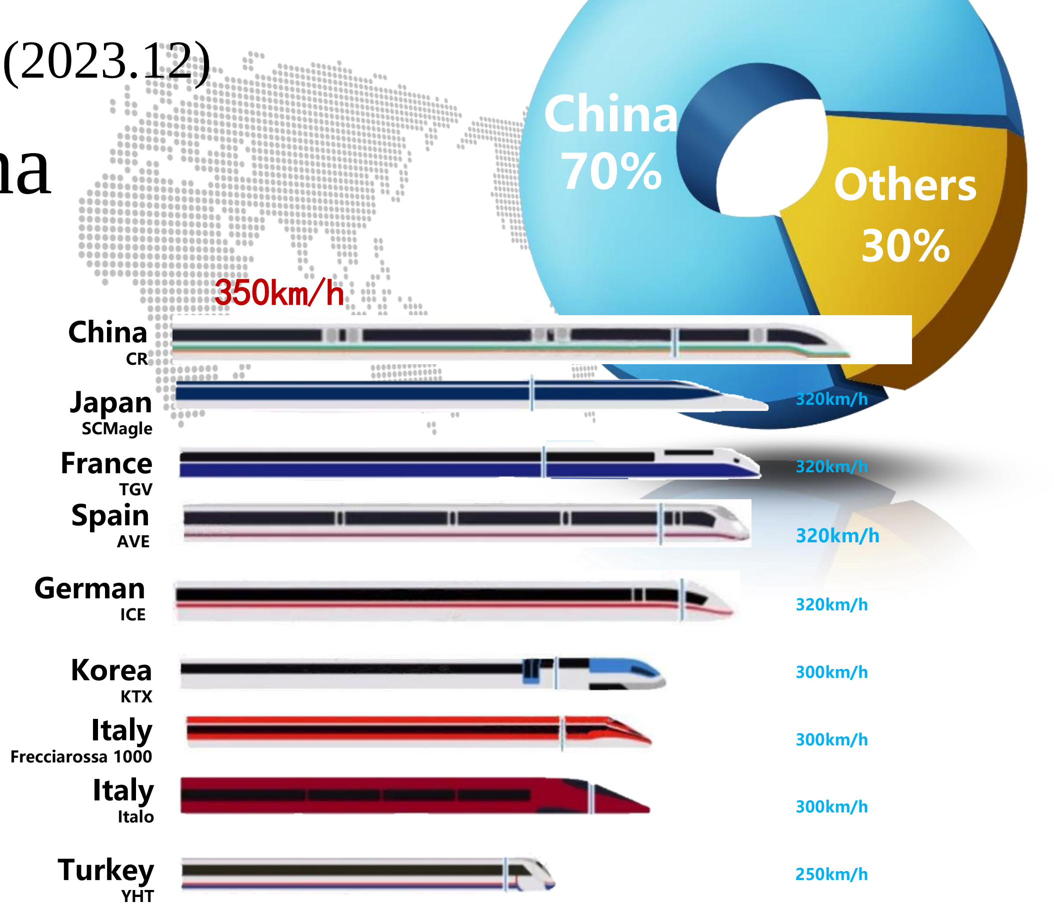
- Introduction
- Model Formulation
- Evolutionary Algorithms for Train Timetable Rescheduling
- Computational Experiments
- Concluding Remarks

Introduction

China High-Speed Railway (HSR)——43700 kilometers (2023.12)

Operation as a network only in China





It is a great challenge to keep the HSR operate punctually

Large network size	High operation speed	High traffic density	Large amount of operation	Complex transportation organization	Diversified travel demand

Train Timetable Rescheduling (TTR) is the key issue for emergency decision under disruption

- If the dispatching is not reasonable, once an emergency occurs, it is easy to cause a large area of train delay and other serious consequences, bringing inconvenience to passengers and reducing the operation efficiency of high-speed railway.

2021.05
Beijing-Tianjin intercity high-speed railway with severe delay since overhead line with foreign matter

2018.12
Heavy snow cause multiple train delay in Changsha South Station

How to propose a simple and effective rescheduling model and a fast solution algorithm has become an urgent need for the efficient operation of high-speed railway.

Train dispatching system is the "brain" and "commander" of high-speed railway system

Real Application

Mainly handled by dispatchers based on their experience under emergencies

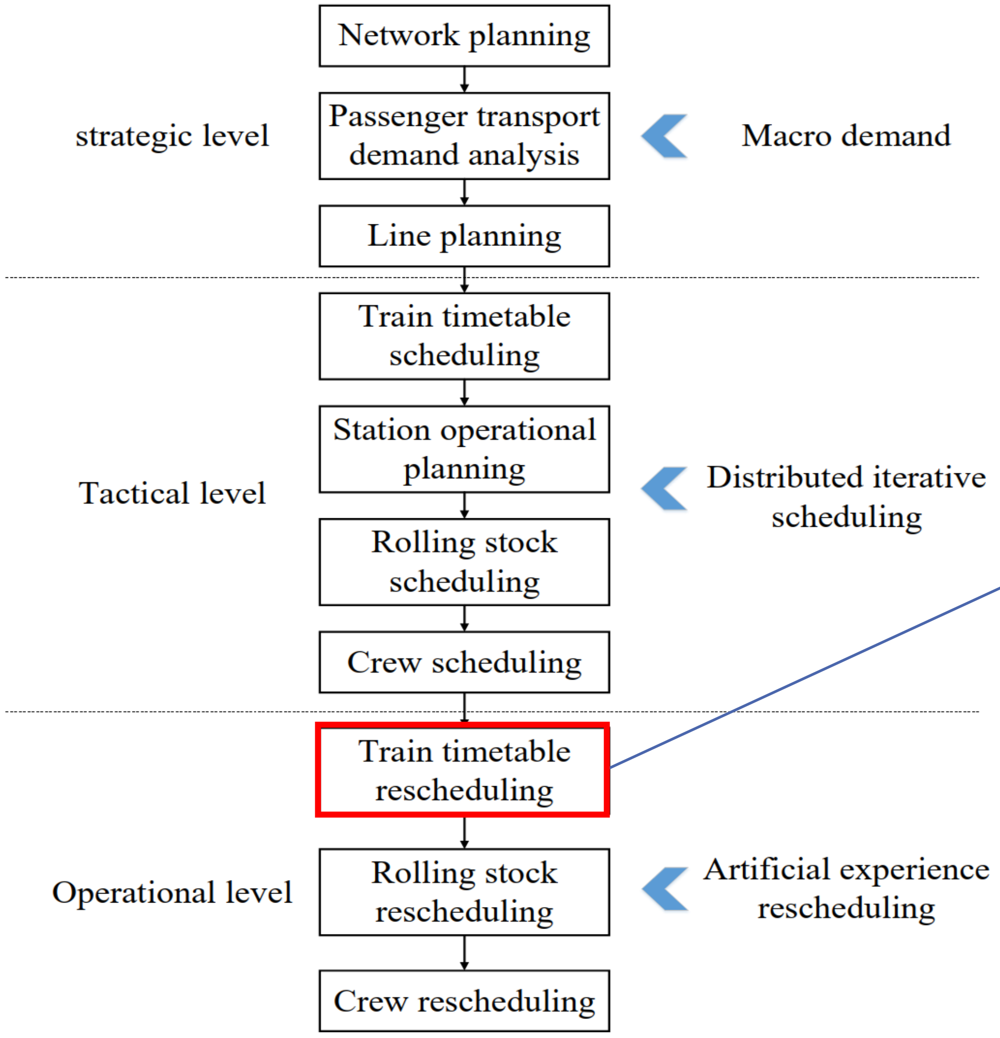
Manual scheduling decision is not optimal decision, which cannot guarantee high efficiency and precise operation

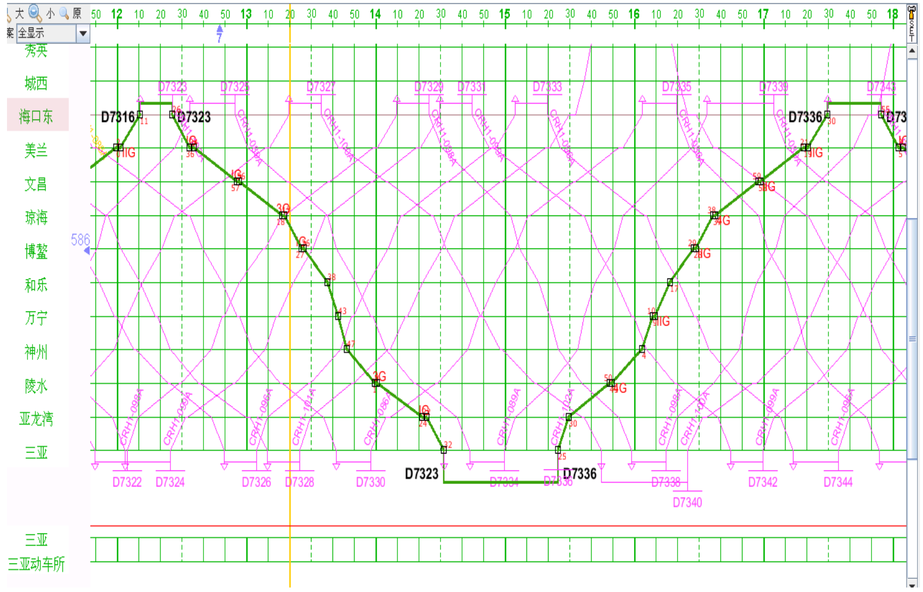
Theoretical research

①Formulate mixed integer linear programming models
②One or multiple objectives
③Use exact method, metaheuristics, or AI technique

① NP-hard
② Time consuming and suboptimal

Different levels in HSR scheduling





Motivation

- HSR may face inevitable emergencies, e.g., infrastructure failure, train failure, natural disasters. Rescheduling is conducted for recovering to normal operation.
- When the scale of the problem is getting larger, and due to multiple objectives, using the CPLEX solver will cost much time, which may exceed the time limit.
- Obtaining the entire Pareto front is time consuming, railway dispatcher only interests in part of the front.

Paper Contribution

- A multi-objective high-speed railway train timetable rescheduling problem with train delay is proposed and modeled as an MILP problem.
- An effective multi-permutation encoding method is proposed for the TTR problem, and a rule-based decoding method is designed to obtain a new schedule. These encoding and decoding methods can manage the entire constraints and guarantee the feasibility of the solution.
- A novel nondominated sorting genetic algorithm-II (NSGA-II) is developed with optimal and suboptimal solutions for initialization and new mechanisms for population crossover and mutation.

Model Formulation

Decision Variables

Symbol	Description
$t_{i,j}^{a}$	the actual arrival time of train i at station j
$t_{i,j}^{d}$	the actual departure time of train i at station j
$q_{i,j,(s,s+1)}$	the actual traversing order, 1 if train i traverses on section $(s,\ s+1)$ before train j; 0 otherwise

$$t_{i,j}^{a}, t_{i,j}^{d} \geq 0 \qquad q_{i,l,k} \in \{0,1\}$$

Formulation

Bi-objective function

- Minimize the total delay time, including the delay arrival and departure time of each train at all the stations
- Minimize the frequency of the train schedule adjustments, calculated by the total number of train arrival/departure time adjustments

$$\min \ F_1 = \sum_{i \in T} \sum_{j \in J} (t_{i,j}^a - T_{i,j}^a) + \sum_{i \in T} \sum_{j \in J} (t_{i,j}^d - T_{i,j}^d)$$

$$\min \ F_2 = \sum_{i \in T} \sum_{j \in J} \mathrm{sgn}(t_{i,j}^a - T_{i,j}^a) + \sum_{i \in T} \sum_{j \in J} \mathrm{sgn}(t_{i,j}^d - T_{i,j}^d)$$

Formulation

Constraints

- Dwell time constraints

$$t_{i,j}^{d} - t_{i,j}^{a} \geq d_{i,j}^{min} \ \forall i \in T; j \in J; (i,j) \notin J^{dis}$$

$$t_{i,j}^{d} - t_{i,j}^{a} \geq T_{i,j}^{d} - T_{i,j}^{a} + d_{i,j}^{dis} \ \forall i \in T; j \in J; (i,j) \in J^{dis}$$

- Running time constraints

$$t_{i,j+1}^{a} - t_{i,j}^{d} \geq r_{k}^{min} \ \forall i \in T; j \in J/\{|J|\}; (i,k) \notin K^{dis}$$

$$t_{i,j+1}^{a} - t_{i,j}^{d} \geq T_{i,j+1}^{a} - T_{i,j}^{d} + r_{i,k}^{dis} \forall i \in T; j \in J/\{|J|\}; (i,k) \in K^{dis}$$

Formulation

Constraints

- Headway constraints for departure headway and arrival headway

$$t^d_{l,j} - t^d_{i,j} \geq H_k q_{i,l,k} - M(1 - q_{i,l,k})$$
$$t^a_{l,j+1} - t^a_{i,j+1} \geq H_k q_{i,l,k} - M(1 - q_{i,l,k})$$

- Traverse order constraint of two trains in a section

$$q_{i,l,k} + q_{l,i,k} = 1 \;\forall i, l \in T; i \neq l; k \in K$$

- Departure and arrival time constraints

$$t^d_{i,j} \geq T^d_{i,j} + d^{dis}_{i,j} \;\forall i \in T; j \in J$$
$$t^a_{i,j} \geq T^a_{i,j} + r^{dis}_{i,k} \;\forall i \in T; j \in J; k = j - 1$$

Formulation

- Model Reformulation
  - Linearization method is developed to deal with sgn(·) in F_2
  - Substitute sgn(·) by

$$\begin{cases} r_1^{i,j} = \mathrm{sgn}(t_{i,j}^a - T_{i,j}^a) \\ r_2^{i,j} = \mathrm{sgn}(t_{i,j}^d - T_{i,j}^d) \end{cases} \quad \forall i \in T; j \in J$$

$$\begin{aligned}
\min \quad & F_1 \\
\min \quad & F_2 = \sum_{i\in T}\sum_{j\in J} r_1^{i,j} + \sum_{i\in T}\sum_{j\in J} r_2^{i,j} \\
\text{s.t.} \quad & M r_1^{i,j} \geq t_{i,j}^a - T_{i,j}^a \ \forall i \in T; j \in J \\
& M r_2^{i,j} \geq t_{i,j}^d - T_{i,j}^d \ \forall i \in T; j \in J \\
& r_1^{i,j} \leq t_{i,j}^a - T_{i,j}^a \ \forall i \in T; j \in J \\
& r_2^{i,j} \leq t_{i,j}^d - T_{i,j}^d \ \forall i \in T; j \in J \\
& r_1^{i,j}, r_2^{i,j} \in \{0,1\} \ \forall i \in T; j \in J \\
& \text{Constraints (3) -- (11).}
\end{aligned}$$

Evolutionary Algorithms for Train Timetable Rescheduling

Encoding and Decoding

- Using multi-permutation-based encoding instead of real-coded encoding
- Real-coded encoding

T: Set of trains J: Set of stations

$$[t^a_{1,1}, t^d_{1,1}, t^a_{1,2}, t^d_{1,2}, \ldots, t^a_{i,j}, t^d_{i,j}, \ldots, t^a_{|T|,|J|}, t^d_{|T|,|J|}], i \in T, j \in J, 1 \le t^a_{i,j}, t^d_{i,j} \le 1440$$

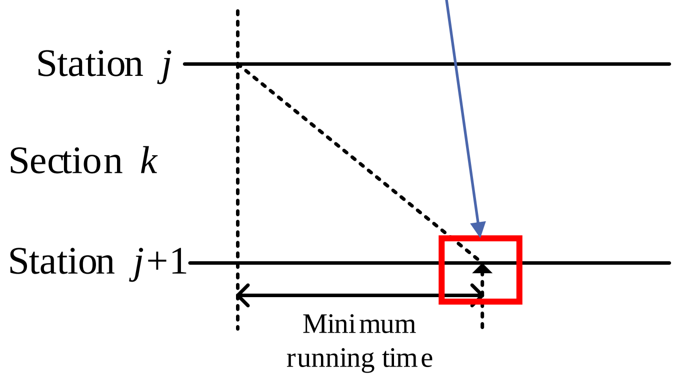
  - Dimension: $2|T||J|$ Solution space: $1440^{2|T||J|}$ (for integer arrival/departure time)
- Multi-permutation-based encoding

$$[p_{1,1}, p_{2,1}, \ldots, p_{i,1}, \ldots, p_{|T|,1}, p_{1,2} \ldots, p_{|T|,2}, \ldots, p_{|T|,|J|}], i \in T, j \in J, p_{i,j} \in \{1, \ldots, |T|\}$$

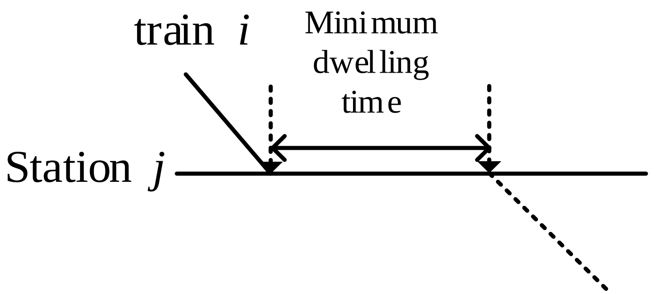
  - Dimension: $|T||J|$ Solution space: $|T|!\ |J|$
- The dimension and solution space is much smaller in permutation-based encoding
- There are unfeasible region in real-coded encoding, constraints handling should be designed

Encoding and Decoding

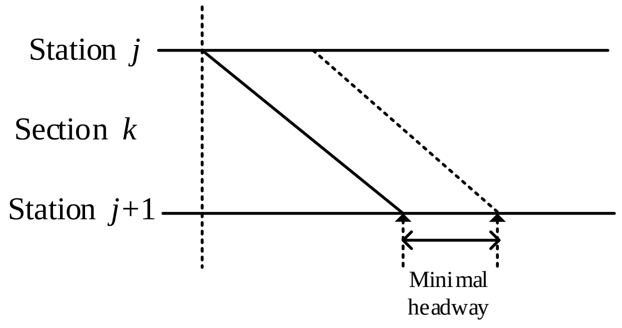
- Obtain the actual arrival time and departure time through the decoding procedure
  - Traversing order is obtained through the permutation-based encoding
  - Decide arrival time and departure time satisfying different constraints



Minimum running time constraints



Minimum dwelling time constraints



Headway constraints

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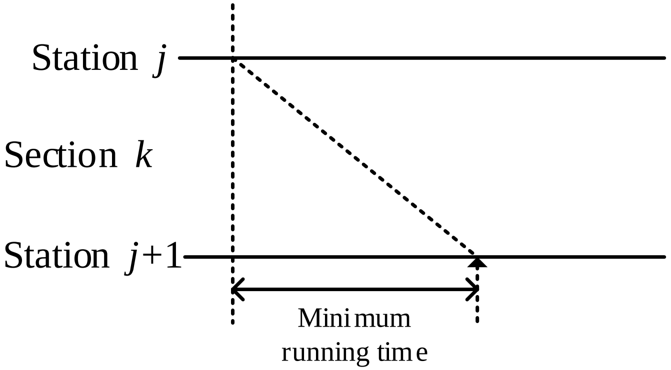

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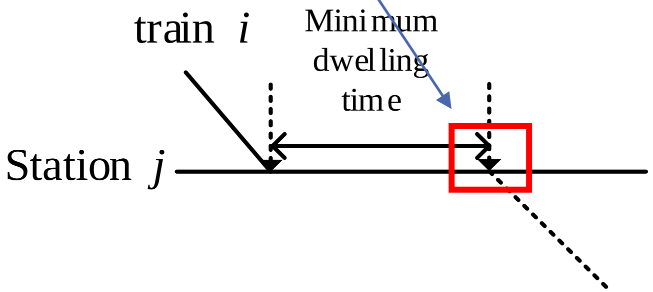

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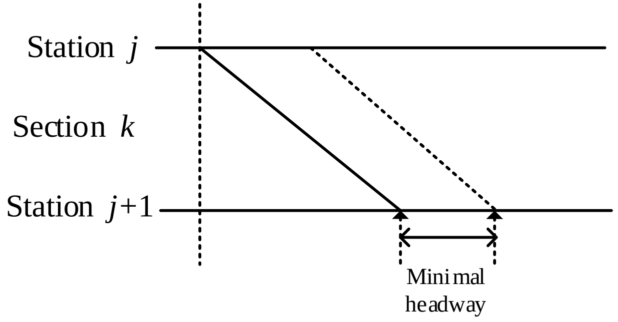

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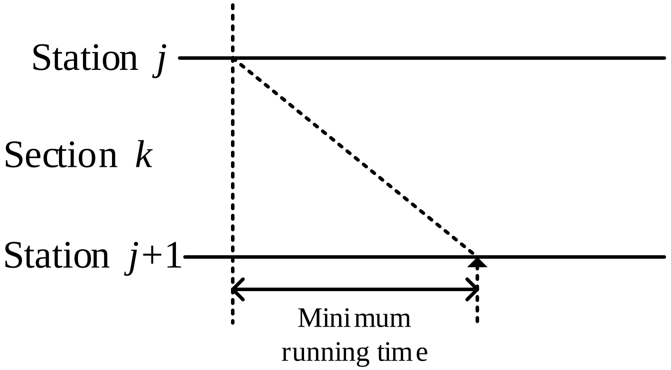

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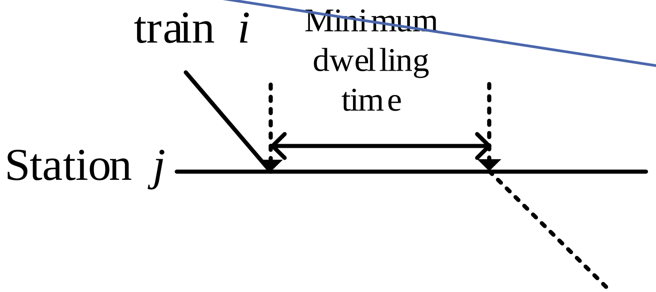

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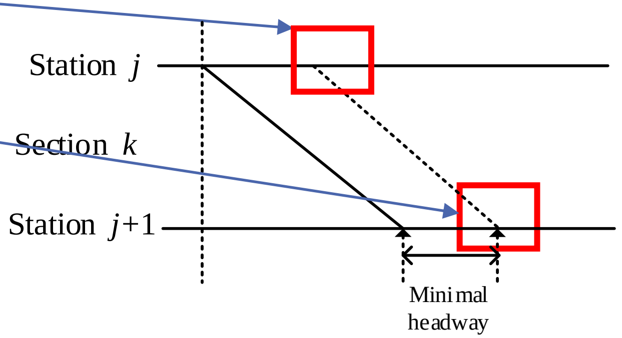


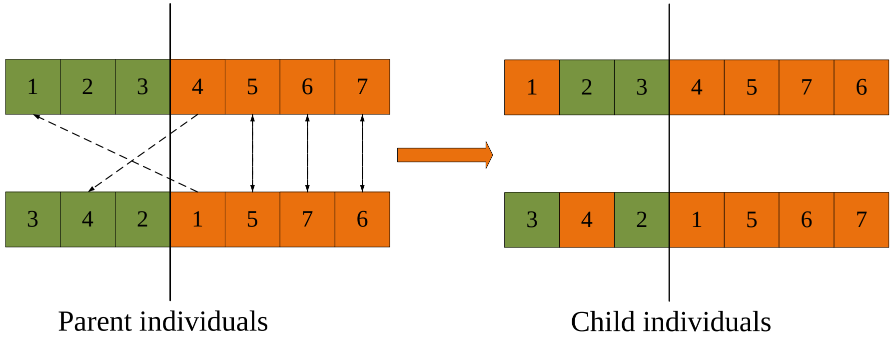
Headway constraints

NSGA-II: Population Initialization

- Random initialization
- Adding one or more Pareto optimal (near Pareto optimal) solutions into the initial population
  - Three optimal solutions with weights weight vector 1 (0.98, 0.02) op1, weight vector 2 (0.2, 0.8) op2, and weight vector 3 (0.02, 0.98) op3
  - One near optimal solution: first-come-first-served (FCFS) strategy (nop)

NSGA-II: Selection, Crossover, and Mutation Operators

- Selection
  - Crowding distance to rank the parent and child individuals within the size of the population
- Randomly select one permutation from $|K|$ permutations for crossover and mutation
- Crossover



Modified order crossover for the permutation-value encoding

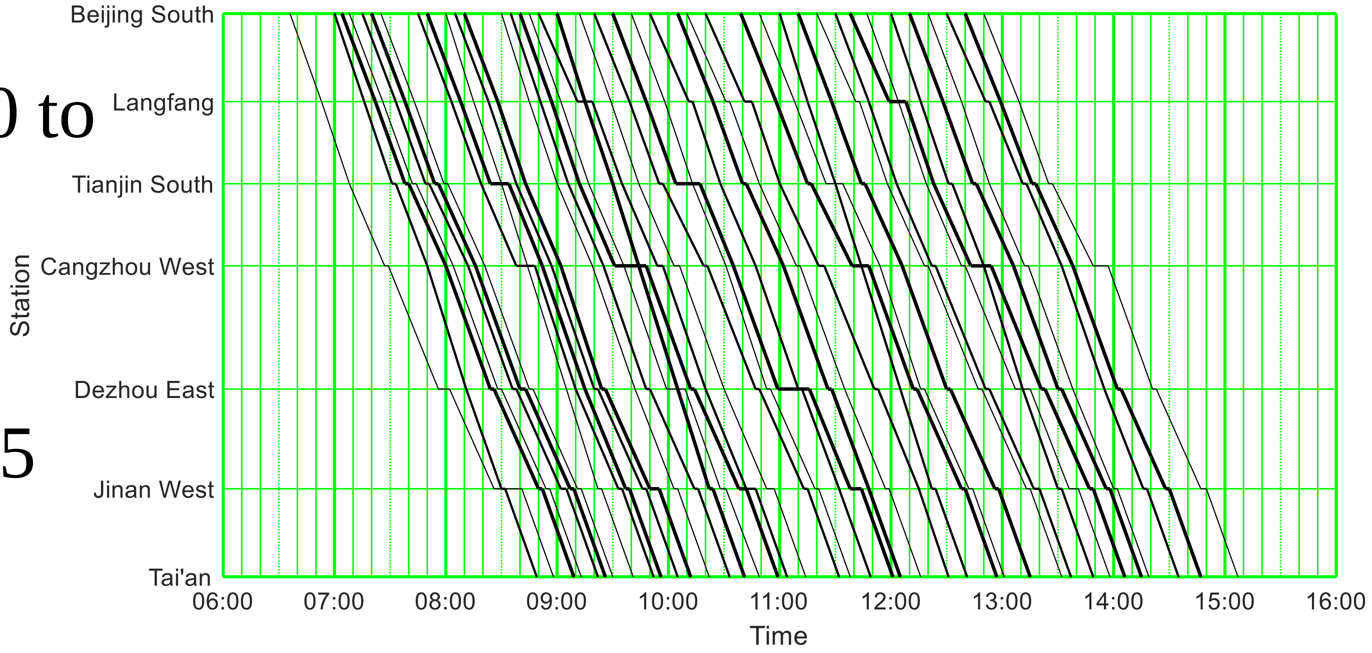
- Mutation

Swap mutation for the permutation-value encoding

Computational Experiments

Computational Experiments

- The Beijing-Tai’an section of Beijing–Shanghai HSR line
- 7 stations and 6 sections
- 40 trains downstream from 6:00 to 16:00
- Dwell time: 2 min
- Minimum running time of each section are 15, 14, 14, 21, 17, 15 (min), respectively
- Minimal headway: 4 min

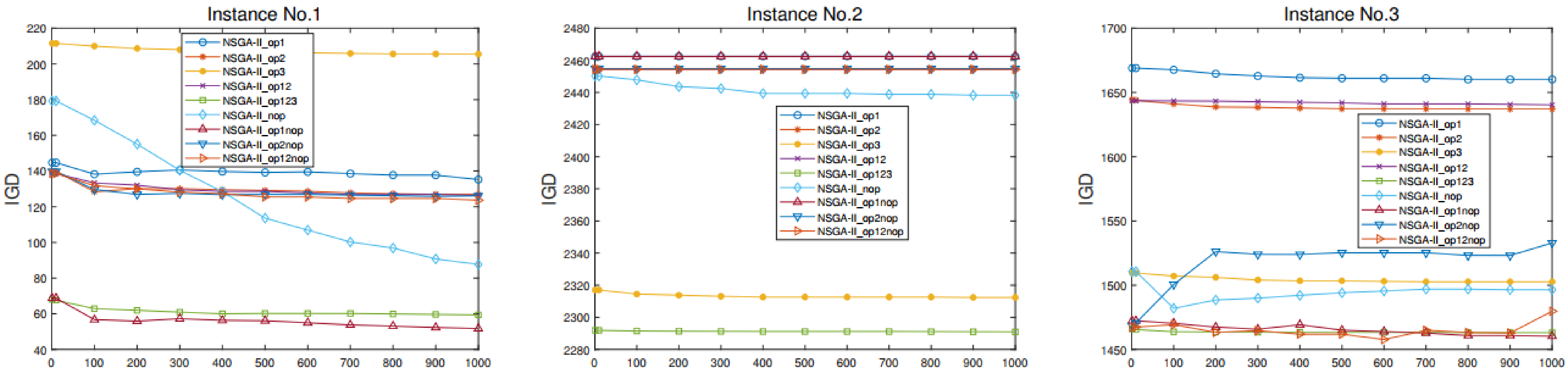


Computational Experiments

- Three test instances
  - Instance No.1: There are only dwell time disturbances when trains stop at stations.
  - Instance No.2: There are only running time disturbances when trains run at sections.
  - Instance No.3: There are both dwell time and running time disturbances.
- Nine subsets of the three Pareto optimal and one near Pareto optimal solutions are used to develop the NSGA-II variants.
  - {op1}, {op2}, {op3}, {op1, op2}, {op1, op2, op3}, {nop}, {nop, op1}, {nop, op2}, {nop, op1, op2}
- Population size $Np = 50$, $MaxGen = 1000$, $p_c = 0.7$, $p_m = 0.3$, 20 independent trials.

Computational Experiments

- The results of the NSGA-II with one or more Pareto optimal (near Pareto optimal) solutions for initialization are better than the original NSGA-II with random initialization. (in terms of IGD)



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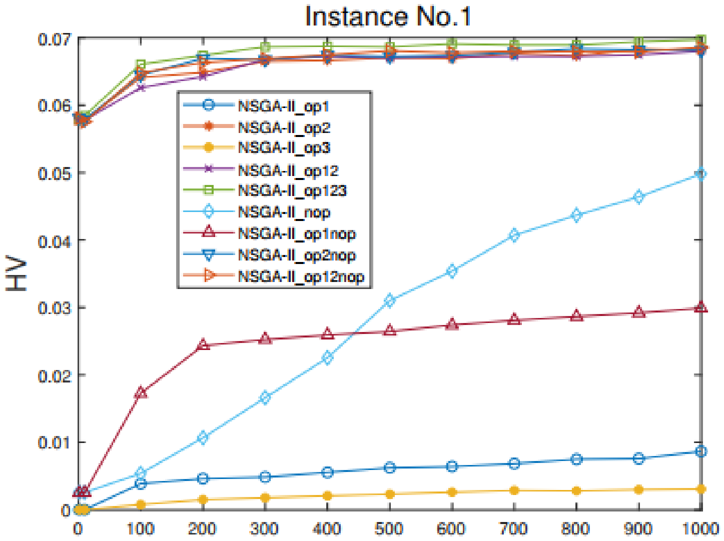
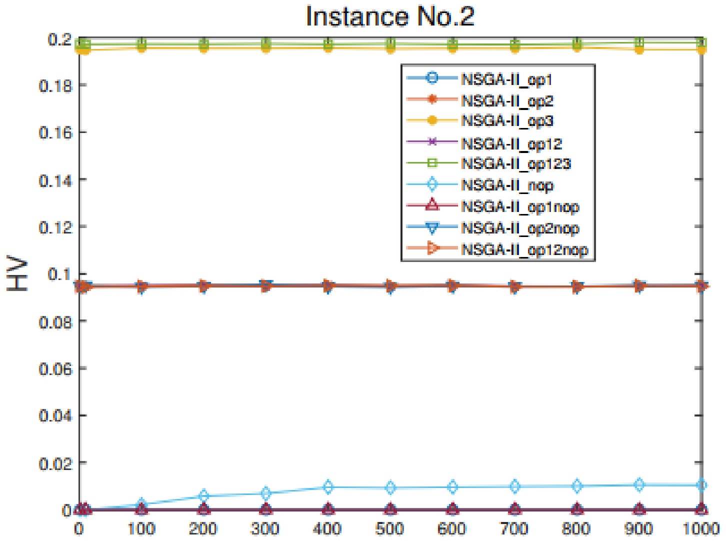
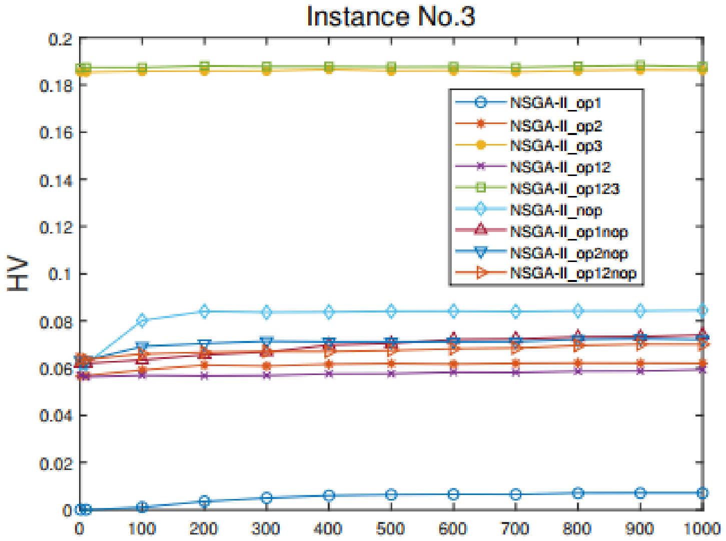
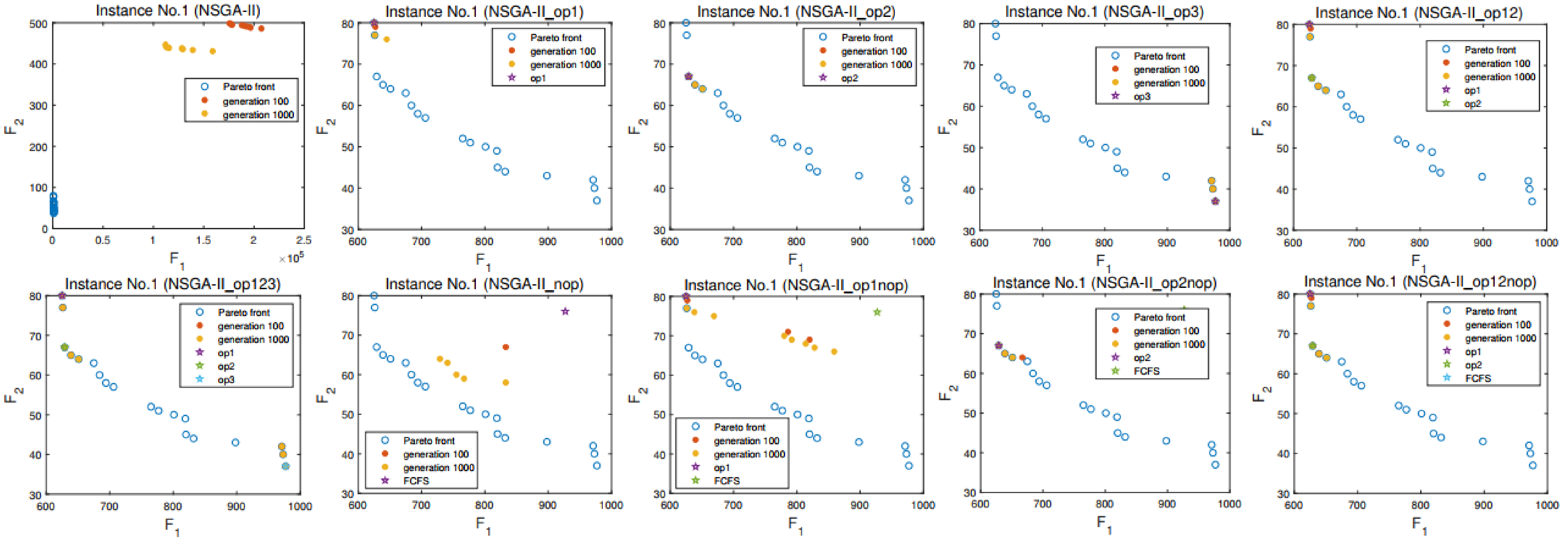






Computational Experiments

- The solutions of NSGA-II are far from the Pareto front on all instances.
- However, if Pareto optimal or near Pareto optimal solutions are included, the obtained solutions are close to the Pareto front and even similar to parts of the Pareto front.



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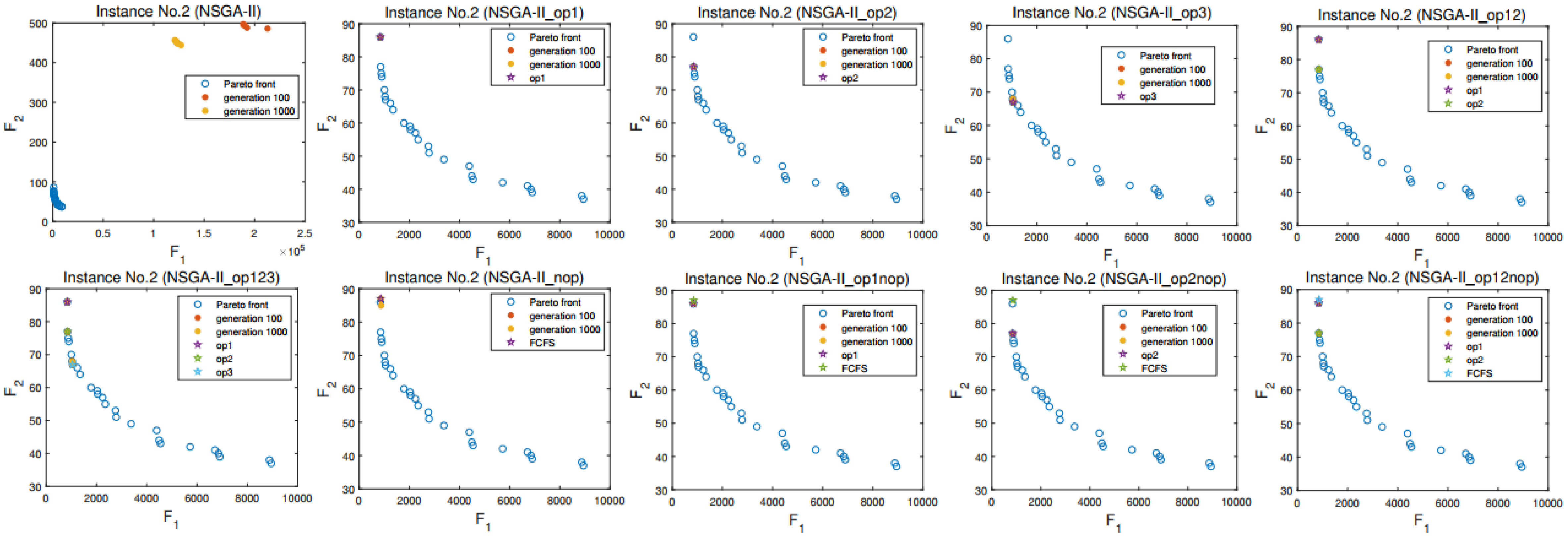

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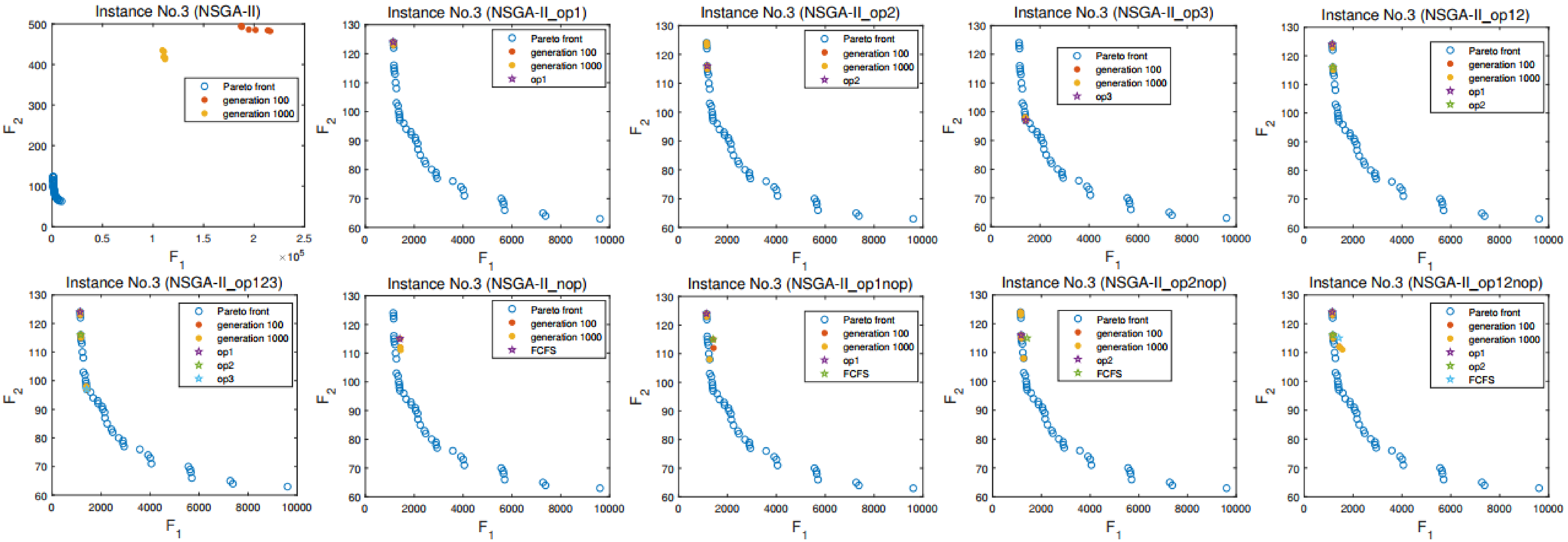

Computational Experiments

- Computation time of NSGA-II (and its variants) on instances Nos. 1-3.
- For instance No.1, the time of the solver may be less than the time by NSGA-II op1, NSGA-II op2, NSGA-II op1nop, and NSGA-II op2nop.
- For instance Nos. 2-3, additional optimal solutions are obtained with less computation time compared with the solver.

TABLE III

RESULTS OF THE COMPARISON ON THE COMPUTATION TIME AND OBTAINED PARETO OPTIMAL SOLUTIONS ON THREE TEST INSTANCES.

Instance		NSGA-II	NSGA-II_op1	NSGA-II_op2	NSGA-II_op3	NSGA-II_op12	NSGA-II_op123	NSGA-II_nop	NSGA-II_op1nop	NSGA-II_op2nop	NSGA-II_op12nop
1	Time (s)	9.42	10.33	11.59	9.82	13.57	15.18	8.35	10.40	11.62	13.58
	# of Optimal Solutions	0	2	3	3	5	8	0	2	3	5
	Time of solver (s)	n.a.	5.50	9.55	13.05	15.05	28.10	n.a.	5.50	9.55	15.05
2	Time (s)	9.75	11.23	11.44	21.43	14.23	27.12	8.58	11.28	11.47	14.36
	# of Optimal Solutions	0	1	1	2	2	4	0	1	1	2
	Time of solver (s)	n.a.	2.61	3.13	44.08	5.75	49.82	n.a.	2.61	3.13	5.75
3	Time (s)	9.78	16.10	20.51	31.95	27.91	51.25	8.66	16.12	20.58	27.99
	# of Optimal Solutions	0	2	4	2	4	6	0	3	5	4
	Time of solver (s)	n.a.	22.98	45.61	72.04	45.61	117.66	n.a.	43.51	66.16	45.61

Concluding Remarks

Concluding Remarks

- The multi-objective high-speed railway TTR problem is formulated as an MILP problem.
- A multi-permutation based NSGA-II is proposed.
- A novel encoding and decoding method are specially designed.
- One or more Pareto optimal and near Pareto optimal solutions are included into the initial population.
- Obtained optimal/suboptimal solutions within one minute.

Future Research

- Develop more efficient operators for NSGA-II.
- Consider other EAs to obtain more Pareto optimal solutions.

Thank you for your attention!